Contracts with Aftermarket Substitution: The Case of PG&E and Electric Batteries

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We study electricity retailer's contract design problem when electric battery providers enter the market for providing behind-the-meter storage service to electricity end-users. Doing this, we also investigate these batteries impact on retailer's profit, its customers' welfare, and consequently battery providers' sales. For a social planner or perfectly informed retailer, batteries have no impact on retailer pricing, and weakly increase all welfares. For rent-seeker retailer with imperfect information, we model the screening problem in a principal-agent setting in which the monopolist retailer offers a menu of consumption bundles and monetary transfers. The incentive compatibility restrictions take into account the agents' option to access battery as a substitution technology in the aftermarket. In the absence of both arbitrage opportunities and dis-economies of scale, agents do not rely on the battery technology in either the first-best or the second-best allocations, implying zero sales for electric battery providers. Furthermore, even with dis-economies of scale, behind-the-meter battery technology reduces the retailer's profit under the second best. We also show that agents' welfare and social welfare may either increase or decrease with batteries.

We empirically study the case of California PG&E, one of the fastest growing markets for behind-the-meter batteries. To this end, we use PG&E data from household hourly electricity consumption. We develop a structural model of household utility by estimating monthly aggregate elasticities from tiered rate households, and cross hour elasticities for substitution from time-of-use rate households. We use unsupervised machine learning to cluster consumers according to their consumption patterns and obtain rich type-heterogeneity. Our empirical results show that behind-the-meter batteries reduce PG&E welfare up to 43%, while they increase its customers' welfare heterogeneously 2% to 400%.

1. Introduction

Electric batteries are rapidly growing technologies that allow storing electricity at one time to be used in another. In recent years, many battery companies have entered the market for end-user batteries. These *behind-the-meter* batteries allow end-users (e.g. households) to store electricity at off-peak prices for use in peak price periods in order to reduce their bills. They also allow arbitrage on retailer's temporal electricity prices.[1] Electricity retailers facing increased use of batteries by end-users should respond by updating their offered prices. By finding the new optimal prices, the retailers would like to understand the impact of end-user battery technology on their profit, as well as their customers' welfare. Finding the retailer optimal contract will also inform behind-the-meter battery providers of their sales.

In this paper, we focus on electricity retailer's optimal electricity pricing considering incentives when batteries are available to the end-users, and study the impact of these batteries on retailer's welfare as well as its customers'. We consider an economy consisting of an electricity retailer, and heterogeneous end-users. The retailer buys electricity in the wholesale market and offers contracts to end-users, while the end-users can access behind-the-meter batteries via battery companies. To isolate the impact of batteries, we consider a monopolist retailer; consequently, we do not consider access to alternative sources of electricity such as rooftop electricity solar panels.

1.1. Theoretical Contribution

We consider the case that the retailer is a monopoly. While electricity market restructuring has promoted competition in some retail markets e.g. in Texas, in many major territories e.g. in California, the retailer forms a monopoly. Theoretically, the monopoly assumption allows isolating the impact of the behind-the-meter batteries in our study by omitting the competition effect across retailers. We show that if the monopoly i) is a social planner or ii) has perfect information of households' types, these batteries do not change the retailer offered contracts, nor they will change the end-users electricity consumption. Furthermore, the end-users will operate the battery only for arbitrage against wholesale prices. Both of these cases are because the optimal retailer pricing will be in the form of passing wholesale prices to end-users with and without the existence of batteries.

Next, and consequent to the first set of results, we consider the case where the retailer is rent-seeker and has imperfect information. This is the case in many electricity retail markets. Rent seeking electricity retailer is the case in many territories including 85% of Europe and 29% of North America (IEA 2020). Moreover, end-users electricity consumption is a factor of many features that are not perfectly observable to the retailers. For example the retailer can not observe the appliance or households lifestyle perfectly. We cast the retailer-end-users problem under above conditions into the principal-agent framework. We model electricity consumption at different hours of a day as a different product. The principal wants to offer a bundle of 24 products to the agents (or in simpler setup just two products for peak and off-peak). We model the substitution technology for end-users (the electric battery) as an aftermarket that allows trading products. We then study

[1] There are also other incentives for end-users to own battery, for example grid independence or green energy, which we do not study in this paper.

the optimal product bundling contract subject to trade in the aftermarket. A contract offered to agents includes the bundle of products and the payment to the principal. The additional trade in aftermarket makes the optimal contract computationally complex. Consequently, the direction of impact on retailer surplus is also not immediately clear.

Our contribution to this screening problem is the following. We prove under the following conditions, no substitution is used by the agents in the aftermarket at the optimal contract: (C1) substitution is cheaper at the retail side than the end-user side, (C2) the aftermarket has economy of scale, (C3) contract space is rich and allows for individual contracts per each type. The intuition behind this *zero-substitution* result is that any agents' substitution in the aftermarket can be endogenized in the contract by the principal at a lower cost. This way the principal can collect the extra surplus that agents want to gain in aftermarket itself, and increase its profit. We should note that contrary to majority of contract design literature, the first order conditions do not hold for this setup, and therefore in the proof we do not rely on those conditions, similar to Di Tella and Sannikov (2021). We provide examples that show the zero-substitution result fails by relaxing (C1), (C2), or (C3).

The zero-substitution result has two immediate implications. First, it implies that the retailer's surplus will weakly reduce under batteries available to end-users. This is because due to zero-substitution results, principal's optimization domain is similar to that without substitution, however, there are extra incentive constraints for agents' deviations. We will show that this implication is true even without condition (C2) and only under conditions (C1) and (C3). Second, the zero-substitution result shows that under Conditions (C1) to (C3), the battery companies have zero sales, but their threat of substitution forces the principal to adjust the contracts offered to the agents. We further discuss this last implication below.

Opposite to our zero-substitution result, in reality several companies sell electric batteries to households. This could be because the assumptions, particularly Conditions (C1) to (C3), do not hold practically. Or it could be because the retailer has not yet adjusted its contract offers in response to this new technology due to for example low penetration rate of behind-the-meter batteries or regulatory frictions. Finally, another reason for positive battery sales could be because of end-users' alternative incentives that are not modeled in our study, for example grid independence, reliability, and preference for green energy. Among the three conditions, (C1) and (C2) are more practical while (C3) may fail in some cases. Condition (C1) holds practically since the substitution cost at the retailer side is determined by the wholesale market prices, otherwise the retailer would have unfulfilled arbitrage opportunity. The wholesale prices in return are lower substitution cost than the end-users battery, otherwise the retailer would again have unfulfilled arbitrage opportunity by passing wholesale prices to end-users. Condition (C2) is valid for the current behind-the-meter battery technologies. However, Condition (C3) may not hold in general for electricity markets. This is because the regulators may limit the retailer in type and number of contracts that are offered to end-users.

Finally to study the impact of batteries on retailer customers, we provide examples that show introducing substitution technology may both increase or decrease agents' surplus even under (C1) to (C3). Moreover this change can be heterogeneous depending on agents' types: while some agents may gain surplus with batteries others may loose. Finally, whenever the increase in agents' surplus overcomes the decrease of the retailer's, the social welfare increases and vice versa.

1.2. Empirical Analysis

Case specific empirical studies complement our theoretical analysis for determining the optimal contract and the magnitude of the impact of behind-the-meter batteries. We present such empirical study for the case of Pacific Gas and Electric Company (PG&E) in California. PG&E territory is a fast growing market for behind-the-meters batteries due to strong state rebate and incentive programs. It also is a close case to our set of assumptions. PG&E is the monopolistic electricity retailer in northern California[2] that aims to maximize its profit subject to the regulatory margin passed by the public utilities commission. PG&E offers non-linear prices which operate for screening end-users based on their unknown types to the PG&E. We focus on households as end-users, and use hourly electricity consumption recorded by smart meter for about 570k households in August 2010 - July 2011 and November 2011 - October 2012 by PG&E.

To solve for the PG&E optimal contracts with end-users battery, we should estimate household behavior under counterfactual prices. Note that we consider the battery impact on household behavior only through bill-saving due to counterfactual electricity prices the batteries induce. To this end, we estimate household utility for electricity consumption. We use a structural approach for modeling household utility which consists of two parts: estimating aggregate monthly consumption (demand elasticity) and the allocation across hours (cross elasticities). For the former we use exponential demand model, and for the latter we use Almost Ideal Demand System (AIDS). For identification of the model parameters, we focus on two groups of households with different tariffs: flat tiered tariff and time-of-use tariff. We use the flat tiered users to identify monthly elasticities; this is because due to lack of price variation across time, flat tarrif ensures no substitution, and therefore the utility function simplifies to one independent of AIDS parameters. We rely on geographical price variation, depending on climate zone, to identify monthly elasticities. Second we use the price variation across peak and off-peak hours for the time-of-use users to estimate the AIDS parameters.

[2] PG&E was a monopoly at the time our data was recorded. Currently, there are Community Choice Aggregators in California but they set their rates following the incumbent PG&E up to 2-3% discount.

In both estimation stages, we face endogeneity due to non-linear tiered prices. We address this endogeneity through two instrumental variables: climate zone which changes the baselines for the tiered prices, and opt in into the CARE program which provides a percentage discount for the households.

Finally, PG&E has imperfect knowledge of households types and its data does not capture all the relevant parameters for household elasticity to electricity prices, for example existence of major appliances or lifestyle. To capture those unobservables, we use unsupervised machine learning to cluster the daily electricity consumption into 9 different clusters, and then attach each household to one of the clusters.

We solve for the optimal PG&E contract Using the estimated model parameters. Since a direct utility function does not exist for AIDS, we use alternative programming by using the notion of *perceived prices*. we solve for the first best, second best without battery and second best with battery. We present the optimal contracts menu with battery for two groups of households with observable parameters prevalent in the data. For each group, the optimal contract menu consists of 9 contracts, one for each cluster of unobservable variables. The resulting contracts follow our theoretical results. They show 14% to 43% reduction in PG&E profit due to behind-the-meter batteries compared to its profit without battery. They also show households' welfare increases between 2% to 400% depending on their type.

1.3. Organization

The rest of the paper is organized as following. We provide a literature review in Section 2. The theoretical analysis for social planner or perfectly informed retailer are presented in Section 3. Section 4 presents the case of rent-seeker retailer with imperfect information including our zero-substitution result. Empirical analysis is provided in Section 5 where we first present data, then type identification with unsupervised machine learning, and structural model. Next, we estimate in two stages the model parameters and solve for PG&E optimal contract with and without batteries. Using these contracts, we discuss empirical results. Conclusion and future directions are provided in Section 6.

2. Literature Review

The related literature to this work are in three parts: impact of electric battery on the grid, theory of contract design, and empirical demand estimation.

2.1. Electric Batteries Impact

There is growing number of studies regarding the impact of electric batteries on the electricity grid and on the electricity market. Some work study the wholesale market when producers gain access to storage. For example Tómasson et al. (2020) studies optimal bidding, and Kirkpatrick (2018) studies market prices and network congestion, Karaduman (2020), Dorsey and Gowrisankaran (2021) study investment equilibrium of battery considering battery's operation in wholesale market, and Dorsey and Gowrisankaran (2021) studies investment on utility-scale battery at equilibrium considering falling battery costs, complementarities with renewable energy, and market power. The value of utility scale battery has been studied both with perfect foresight of future prices (Sioshansi 2011, Sioshansi et al. 2009) and considering uncertainty of future prices (Mohsenian-Rad 2015, Xi et al. 2014). Batteries available to retailer are studied in Rasouli et al. (2019), Liu et al. (2017), Zhao et al. (2019), where they show the retailer has multiplexing gain by giving battery to multiple end-users. Finally, several papers study the value of battery for end-users both for reducing cost of electricity (bill saving) (Li 2019, Setlhaolo and Xia 2015, Park and Lappas 2017, Hanna et al. 2014, Patel et al. 2017) and for avoiding power outage (Laws et al. 2018, Tsianikas et al. 2019, Tostado-Véliz et al. 2021). To the best of our knowledge we are the first to study the impact of the end-user batteries on the retailer's contract design.

2.2. Contract with Substitution

Following seminal work of Myerson (1981), there have been several papers on the monopolist screening through contract design. Recently there has been work on contracts with aftermarkets where the agents deal with strategic third party after they finalize the contracting with the principal (Dworczak 2020). In our work the third party aftermarket is non-strategic, in other words battery provider companies offer their services at competitive price. Another relevant work to substitution technology is a saving account available to agents. Di Tella and Sannikov (2021) considers dynamic contracts where agents have hidden saving as a substitution device. Similar to the work in Di Tella and Sannikov (2021), the first order conditions can not be used for our setup, and consequently, we use alternative methods.

Contract design specific to electricity retailers is studied in the work of Joskow and Tirole (2006), where it is shown when the retailer has either perfect information or is social planner, the optimal prices take a Ramsey form. Here the retailer passes wholesale prices to agents. We first extend their results to the case where households have battery. Next, and more importantly, we study the case where the retailer has both imperfect information and is rent-seeker. In this case Ramsey prices are not optimal. Our zero substitution result however shows the optimal contract does not involve any substitution.

2.3. Electricity Demand Estimation

Electric Power Research Institute (EPRI) (2013) is a general guideline and summary of studying household electricity demand. Many work focus on estimation of monthly electricity consumption with respect to tiered electricity rate. Ito (2014) uses data from monthly residential electricity consumption data at the border of two retailer territories, and argues that households respond to average price instead of the marginal price. The identification uses a difference-in-difference method which takes the first-difference with respect to the same billing month of the previous year. Ito (2010) uses the same identification as Ito (2014) except for using the interaction of nonparametric consumption control and electricity service area indicator as the instrument.Reiss and White (2005) assesses the household electricity consumption elasticities using annual aggregate data of 1307 households under a two-step tariff in California from 1993 to 1997. The authors also have acccess to both appliance survey and demographic survey. They also explore heterogeneity in household elasticity not just with respect to their consumption level, but also to their appliance holdings. Carter et al. (2012) estimated a demand model under three-tier pricing schedule and uses the estimated model to examine the impact of tariff changes in a representative sample of 130 Barbadian households. The authors also explore the elasticity heterogeneity in the appliance owned by the households. Similar to Ito (2014), our first stage estimation for aggregate monthly demand also leverages the spatial price variation, but the spatial price variation in our study comes from different climate zones served by a single retailer.

The estimation of electricity demand spread across different hours of the day is also investigated in the literature. Ata et al. (2018) provides an empirical analysis of the impact of retail time-based tariffs on peak load, household's bill savings and carbon emissions. The electricity consumption and survey data of $3,412$ households in Ireland representative of the entire population of residential households are used for estimation. Allcott (2011) studies the impact of real-time pricing on household electricity consumption by a random experiment run in Chicago in 2003. Reiss and White (2008) measures the impact of real-time pricing on household demand by non-experimental data of the California crisis. Davis (2008) argues that time-intensive consumption, e.g. washer or oven, have little price elasticity from efficient devices but it does not address the intra-day shifts. Andersen et al. (2017) studies intra-day shift in electricity. The estimation is done within the total 737 households signed up for the rebate based program and 1,065 households signed up for the GHG-free energy program. Households were prompted via text message to their cell phones a few hours in advance on the same day they were supposed to move power. In our study we use households under time-of-use tariff for estimating substitutions (cross-elasticities) using AIDS model.

The AIDS model used in our second stage is proposed by Deaton and Muellbauer (1980). Similar to many other studies we use the linear approximation of the AIDS (LA-AIDS) using the Stone price index following Blanciforti and Green (1983). Filippini (1995a) and Yang et al. (2014) have estimated AIDS model for households electricity consumption under ToU pricing and real-time pricing, respectively.

To the best of our knowledge, we are the first to combine the estimation of both electricity monthly demand and cross elasticities based on a two-stage estimation, and estimate utility function for hourly demand. A two-stage demand estimation similar to ours, where agents in the first stage choose their total expenditure on a bundle of products and then decide how to divide it across those products, is studied in other areas. For example, Chaudhuri et al. (2006) studies this model within the context of patent collection. Wang and Çakır (2021) estimates a two-stage structural demand system for consumption patterns of cereals in Ethiopia. Menezes et al. (2008) estimates demand elasticities for food products in Brazil using a two-stage budgeting system. Fan et al. (1995) studies the demand system of Chinese rural households by estimating a two-stage LES-AIDS model.

3. Social Planner or Fully Informed Retailer

In this Section we show that unless the principal is rent-seeker and has imperfect information of agents' types, substitution available to end-users does not affect the pricing offered by retailer. Furthermore, all welfare weekly increase in those cases.

PROPOSITION 1 **(No Battery Effect)**. *If the retailer is social planner or has perfect information of the end-users types, behind-the-meter batteries do not impact retailer optimal pricing.*

Proof. For a social planner retailer, the optimal pricing is passing the wholesale prices directly to end-users. This will not change with batteries.

For the fully informed retailer, the optimal pricing is passing the wholesale prices directly to the end-users and collecting all of the type-specific surplus. The pricing does not change with battery although the collected surplus changes. □

REMARK 1 (ZERO BATTERY USE). From Proposition 1, for a perfectly informed retailer or a social planner retailer if the battery cost for end-users gives no arbitrage opportunity in the wholesale market, then end-users have zero battery use.

REMARK 2 (WELFARE INCREASE). Welfare of a perfectly informed retailer will increase with the batteries. This is because from Proposition 1 the retailer fully collects end-users surplus, and on the other hand the end-users surplus increases with battery. The latter is true because the offered prices remain the same as wholesale prices with or without battery, and therefore the battery, as an additional substitution capacity, can only increase end-users surplus from using electricity. In this case, agents surpluses after payment to retailer remain at zero value. Consequently social welfare will increase with battery.

For a social planner, the retailer's welfare remains at zero value with battery. This is because from Proposition 1 the retailer just passes wholesale prices to end-users. However, the agents' welfare and consequently social welfare increase with battery.

4. Rent-Seeker and Imperfectly Informed Retailer: Screening with Substitution Aftermarket

In this section we consider the case of rent-seeker retailer with imperfect information.

4.1. Model

We provide a principal-agent model for the retailer's problem of electric batteries entering the market.

Consider a population of agents (electricity end-users), and principal (electricity retailer). Agents use a bundle of products (electricity consumption per each time period, for example each hour in 24 hours). The principal offers contract in the form of a bundle of products and a payment for that bundle. Principal has production cost (buying or selling electricity in the wholesale market). Agents can gain access to a substitution device (electric battery). We now model each player of this economy in more detail.

Agents are of types $\theta \in \Theta$. Agents buy/sell product bundle d from/to principal. d is a vector such that its i-th element denotes the planned consumption of good i. $\mathcal{D}$ denotes the set of all trade plans with principal that the agents can choose from. Agents have access to substitution device that allows a vector of substitution $x \in \mathcal{X}$ at cost $b^a(x)$ where x is a vector and $b^a(x)$ is a scalar. $d + x$ is agent's consumption vector.

Remark 3. $\mathcal{D}$ and $\mathcal{X}$ denote all feasible consumption of the agents as well as technological limits. For example, to encode behavioral restrictions of households to change their electricity consumption rapidly, $\mathcal{D}$ should allow for only limited changes in consumption across products.

Let $U_\theta(d + x)$ be a general utility function that agent type θ derives from consumption plan $d + x$. Assume quasilinear preferences.

The principal offers a menu of contracts to agents. Principal's cost of producing product vector y is $\tilde{b}(y)$. In the case of electricity retailer $\tilde{b}(y)$ is the buy/sell cost at the wholesale market. Each contract offered by retailer to agents is of the form $C = (c_\theta, \theta \in \Theta) = (D, T)$. $D = (d_\theta, \theta \in \Theta)$, where $d_\theta \in \mathbb{R}^T_+$ is the consumption bundle offered to type θ by the principal, and $T = (t_\theta, \theta \in \Theta)$, $t_\theta \in \mathbb{R}$ is the amount charged to type θ. We denote the space of all such contracts that the retailer can offer by $\mathcal{C}$. The principal offers a menu of contract to agents denoted by M, where $M \subset \mathcal{C}$. $\mathcal{M}$ denotes the space of all menus available to the principal.

Remark 4. The contract form studied here are the most general form of the contracts. In other words, for every other form of contracts such as Ramsey pricing there exists a contract of $C \in \mathcal{C}$ that induces the same agents' consumption of electricity and payments to the retailer.

Remark 5. $\mathcal{C}$ and $\mathcal{M}$ incorporate the structural limits on the contracts offered to the agents by regulator, for example contracts only on Ramsey forms. There could be other regulatory or practical limits for example on the maximum number of different contracts $C \in \mathcal{C}$ that are offered to agents to choose from. Such limits can be captured by $\mathcal{M}$.

The timing of the events is as follows. First, principal chooses the contract menu M to offer to agents, and then agents take their action. The action of each agent type θ is to choose the pair (c_θ, x_θ) where $c_\theta \in M$ and $x_\theta \in \mathcal{X}$; here c_θ is the contract agent chooses and x_θ is the agent substitution in the aftermarket. Denote action space with A. Without access to substitution device, agents' action is limited to $(c_\theta, 0)$.

The optimal action for agents of type θ facing contract menu M is

$$c^*_\theta(M) \in \arg\max_{c \in M} U_\theta\big(d + x^*_\theta(c)\big) - t - b^a\big(x^*_\theta(c)\big) \tag{1}$$

where

$$x^*_\theta(c) \in \arg\max_{x \in \mathcal{X}} U(d + x) - b^a(x). \tag{2}$$

Note that $c^*_\theta(\cdot)$ and $x^*_\theta(\cdot)$ are slightly abused here as the optimal contract selected by type θ and the optimal substitution facing a contract, respectively.

The optimal contract design problem of the principal is for the principle to maximize its surplus and the way we formulate the optimization is to let the contract designed for type θ to be the optimal action for agents of type θ, that is, $c^*_\theta(M) = c_\theta$. The problem is formulated with the following constrained optimization problem

$$M^* \in \arg\max_{M \in \mathcal{M}} \sum_{\theta \in \Theta} \pi_\theta [t_\theta - \tilde{b}(d_\theta)]$$

$$s.t. \quad U_\theta\big(d_\theta + x^*_\theta(c_\theta)\big) - t_\theta - b^a\big(x^*_\theta(c_\theta)\big) \geq U_\theta\big(d_{\theta'} + x^*_\theta(c_{\theta'})\big) - t_{\theta'} - b^a\big(x^*_\theta(c_{\theta'})\big) \quad \forall \theta \in \Theta, \theta' \in \Theta - \{\theta\} \quad IC \tag{3}$$

$$U_\theta\big(d_\theta + x^*_\theta(c_\theta)\big) - t_\theta - b^a\big(x^*_\theta(c_\theta)\big) \geq 0 \quad \forall \theta \in \Theta \quad IR \tag{4}$$

Here π_θ is the fraction of users in type θ.

We are now ready to present the problem of screening with substitution.

4.2. Zero Substitution

We first present conditions for the zero substitution results and then present the results themselves.

4.2.1. Zero Substitution Conditions We use conditions (C1) to (C3) discussed in the introduction for the results in this section. We define them more rigorously here.

Definition 1. Conditions (C1) to (C3) are the following:

- (C1) No arbitrage: principal has lower cost of substitution than agents, $b^a(x) \geq b^p(x)$ for all x.
- (C2) Economy of scale in aftermarket: for all $x, z \in \mathbb{R}^n$:

$$b^a(x) + b^a(z) \geq b^a(x + z) \tag{5}$$

- (C3) Rich contract space: principal can offer a large menu of contracts consisting of one contract per agent without any structural limit on the shape of the contract.

We discuss conditions (C1) to (C3) from practical stance point. (C1) is often practically valid because the principal often has access to the same if not more complicated technologies. In the specific case of electricity, first the principal substitution cost is equal to the arbitrage price in the wholesale market. This is because if the substitution cost for principal is cheaper than wholesale market arbitrage, the principal can arbitrage in the wholesale market using its substitution technology and this way reduce arbitrage price in the wholesale market. On the other hand the principal substitution cost can not be higher than the wholesale market arbitrage price, because then the principal can use wholesale market for substitution. Second, if the substitution by agents in cheaper than principal, then the principal can arbitrage through buying electricity from wholesale market, passing to the agents to substitute for cheaper price, and buy back fro agents and sell back to the wholesale market to the wholesale market to the point that the cost of arbitrage is the same with wholesale market.

(C2) or the economies of scale condition is equivalent to subadditivity of the cost function $b^a(\cdot)$. This property holds when $b^a(\cdot)$ is a norm because subadditivity is implied by the triangular inequality. In particular, the functions $b^a(x) = \phi \sum_l |x_l|$ and $b^a(x) = \phi \sum_l \sqrt{|x_l|}$ satisfy the economies of scale condition. Particularly, (C2) holds for the special case of electric batteries because we have two conditions determining $\mathcal{X}$. First no storage beyond the available endowment (no purchase of extra electricity for storage) implies

$$-d_t \leq x_t, \forall t \tag{6}$$

Second no negative storage implies

$$x_0 \leq 0 \tag{7}$$

$$x_t \leq -\sum_{1}^{t-1} x_\tau, \qquad \forall t > 0 \tag{8}$$

Also, the agent substitution costs are in the form of

$$b^a(x) = f(\max_t \sum_{1}^{t} -x_\tau) \tag{9}$$

where f is the cost function for purchasing a battery of certain capacity. This cost function implies a simplified assumption that the agent substitution cost is a function of power capacity rather than energy capacity (taking battery as an example).

(C3) does not hold in some markets. Particularly in electricity sector retailers are limited to a few contracts and maybe in the specific form of tiered pricing. Relaxing (C3) by posing limits on contract forms is a future research direction, and we do not consider it in our first attempt to the problem in this paper.

REMARK 6. The model can easily accommodate feasibility restrictions on the substitution technology. For batteries, it would be natural to restrict the feasible set to $\mathcal{X}_{battery} = \{x : \sum x_t = 0\}$ or, equivalently, to set $b^a(x) = \infty$ for all $x \notin \mathcal{X}_{battery}$. Condition (C2) implies that the feasibility set $\mathcal{X}$ has to satisfy the following unlimited substitution condition:

$$\{y + x | y \in \mathcal{X}\} \subseteq \mathcal{X} \quad \forall x \in \mathcal{X} \qquad \text{(unlimited substitution)}$$

To see this, suppose that the original model is $(b^a, \mathcal{X})$ where $b^a : \mathcal{X} \to \mathbb{R}$ is the cost of substitution and $\mathcal{X} \subset \mathbb{R}^n$ is the substitution space. Define an alternative but equivalent model $(\tilde{b}, \mathbb{R})$ where $b'^a(x) = b^a(x)$ for all $x \in \mathcal{X}$, and $b'^a(x) = \infty$ for all $x \notin \mathcal{X}$. Conditions (C2) and unlimited substitution in the original model are equivalent to condition (C2) in the alternative model.

If the alternative model satisfies (C2), the original model will satisfy both (C2) and unlimited substitution. The proof is simple. Take any $x, y \in \mathcal{X}$, (C2) implies that $b'^a(x) + b'^a(y) \geq b'^a(x + y)$ is finite; thus, $x + y \in \mathcal{X}$. Moreover, $b^a(x) + b^a(y) \geq b^a(x + y)$.

Unlimited substitution holds when $\mathcal{X}$ is unbounded in all dimensions that are feasible for substitution at least in one direction. Particularly it holds for both unlimited unidirectional and unlimited multi-directional substitution.

4.2.2. Zero Substitution Results We are now ready to present our major results.

PROPOSITION 2 **(Second Best Zero Substitution)**. *Under conditions (C1) to (C3), there exists an optimal contract offered by principal for which the agents do not use substitution.*

Proof. The argument of this Proposition is that there exits an optimal contract for which $x^*_\theta = 0$ in aftermarket for all $\theta \in \Theta$. We show this by contradiction. Suppose we have an optimal contract menu M where the contract for type θ, c_θ, enforces agent θ to choose its optimal action $x^*_\theta(c_\theta) \neq 0$. We prove by constructing a weakly preferred contract menu M' for the principal that ensures $x^*_\theta(c'_\theta) = 0$ while it does not change the decisions of agents of other types. We thus construct the menu M' such that contracts for types other than type θ satisfy $(d'_{\overline{\theta}}, t'_{\overline{\theta}}) = (d_{\overline{\theta}}, t_{\overline{\theta}})$ for all $\overline{\theta} \in \Theta \backslash \{\theta\}$, while $d'_\theta = d_\theta + x^*_\theta(c_\theta)$, and $t'_\theta = t_\theta + b^a(x^*_\theta(c_\theta))$. We show that $x^*_\theta(c'_\theta) = 0$, M' satisfies IR and IC constraints for all types of users, and M' brings higher surplus for the principle.

First, based on (C2), if for $c_\theta = (d_\theta, t_\theta)$ we have $x^*_\theta(c_\theta) \neq 0$, then for $c'_\theta = (d_\theta + x^*_\theta(c_\theta), t_\theta + b^a(x^*_\theta(c_\theta)))$, we have $x^*_\theta(c'_\theta) = 0$. Second, it is clear that IR and IC constraints still hold for type θ as its agent surplus under contract c'_θ has not changed. For all $\overline{\theta} \in \Theta \backslash \{\theta\}$, $IR_{\overline{\theta}}$ does not change under $c'_{\overline{\theta}}$ either and therefore remains valid. It remains to show $IC_{\overline{\theta}}$ holds under $c'_{\overline{\theta}}$ for all $\overline{\theta} \in \Theta \backslash \{\theta\}$.

$$U_{\overline{\theta}}(d_{\overline{\theta}} + x^*_{\overline{\theta}}(c_{\overline{\theta}})) - t_{\overline{\theta}} - b^a(x^*_{\overline{\theta}}(c_{\overline{\theta}})) \qquad (10)$$

$$
\begin{aligned}
&\geq \max_{x\in\mathcal{X}} U_{\overline{\theta}}(d_\theta + x) - t_\theta - b^a(x) && (\ IC_{\overline{\theta}} \text{ under } M)\\
&= \max_{x\in\mathcal{X}} U_{\overline{\theta}}(d'_\theta - x^*_\theta(c_\theta) - x) - t_\theta - b^a(x) && (\text{by plugging in } d'_\theta)\\
&= \max_{x\in\mathcal{X}} U_{\overline{\theta}}(d'_\theta - x^*_\theta(c_\theta) + x) - t'_\theta + b^a(x^*_\theta(c_\theta)) - b^a(x) && (\text{by plugging in } t'_\theta)\\
&\geq \max_{x\in\mathcal{X}} U_{\overline{\theta}}(d'_\theta - x^*_\theta(c_\theta) + x) - t'_\theta - b^a(x - x^*_\theta(c_\theta)) && (\text{due to (C2)})\\
&= \max_{y\in\mathcal{Y}} U_{\overline{\theta}}(d'_\theta + y) - t'_\theta - b^a(y) && (11)\\
&\geq \max_{x\in\mathcal{X}\subseteq\mathcal{Y}} U_{\overline{\theta}}(d'_\theta + x) - t'_\theta - b^a(x) && (\text{due to unlimited substitution})
\end{aligned}
$$

where the last line is the $IC_{\overline{\theta}}$ under $c'_{\overline{\theta}}$. Third, due to (C1) we have $b^p(x^*_\theta(c_\theta)) \leq b^a(x^*_\theta(c_\theta))$ and therefore M' gives the principle higher surplus than M. Note that the above construction is using rich contract space from (C3) that allows changing one agent's contract without change to the others'.□

Remark 7. In Proposition 2, we assumed the substitution cost of every agent, $b^a(.)$, is only a function of his own demand for substitution in the aftermarket. In practice, the substitution costs can depend on all other agents' substitution, for example due to elasticity of prices in the battery market. It is straightforward to see the proof of Proposition 2 holds even if the cost in the aftermarket also depends on other agents' decisions, $b^a(x_\theta, x_{-\theta})$, as far as it has decreasing slop of supply (economy of scale in total supply).

Proposition 2 shows the principal can solve a relaxed problem for finding the optimal contract by setting agents' substitutions to zero. This in turn simplifies the principle's program. Furthermore, this proposition shows although the threat of substitution by agents forces the principle to change its contracts, the substitution will not be realized at agents' side under conditions of the proposition. And immediate implication of this result is that battery providers to end-users do not sell any battery under retailer's best response.

Proposition 2 also opens the question of why practically such substitution devices are sold in the market by different companies, e.g. electric batteries. One answer could be conditions (C1) to (C3) are violated which we study in Lemma 1 below. Other answers could be the principal has not updated its contract to respond to existence of such substitution technologies. Finally, agents may have other incentives (e.g. grid independence in the case of electric battery) that is not modeled in this study.

Lemma 1 shows the results of Proposition 2 are sensitive to the conditions (C1) to (C3).

Lemma 1. *In Proposition 2, if (C1), (C2) or (C3) fail, then agents may use aftermarket substitution at optimal contract offered by the principal.*

Proof. It is straightforward to see if (C1) fails, the principal will rely on agents to substitute with lower cost and collect the surplus. For relaxing Conditions (C2) and (C3), see Example 1 in Appendix A, Cases 1 and 2 correspondingly.□

4.3. Welfare Analysis

Before studying the welfare impact, we make the following remark about the first best results.

COROLLARY 1 **(First Best)**. *Ignoring incentives, substitution accessible to agents will not change the first best solution under condition (C1). Therefore, substitution device does not have any welfare impact under the first best condition.*

REMARK 8. If conditions (C1) to (C3) hold, then an immediate consequence of zero-substitution result in Proposition 2 is that the principal's surplus decreases with agent substitution, since the space of contracts is the same as those without agent substitution but with more strict IC constraints. However, the following Proposition 3 shows this result is in fact independent of condition (C2).

PROPOSITION 3 **(Second Best Principal's Surplus)**. *Under conditions (C1) and (C3) principal's surplus weakly decreases when agents have access to substitution device.*

Proof. For the optimal contract $c_\theta = (d_\theta, t_\theta)$ with agent substitution $x^*_\theta(c_\theta)$. Note that $x^*_\theta(c_\theta) = 0$ may not hold as (C2) no longer holds in this setting. We prove by constructing a contract c'_θ for all $\theta \in \Theta$ that is implementable without substitution technology available to the agents but gives equal or higher surplus to the principal.

Consider contract $c'_\theta = (d_\theta + x^*_\theta(c_\theta), t_\theta + b^a(x^*_\theta(c_\theta)))$ without agent substitution. IR constraint for type θ under contract c'_θ holds because

$$U_\theta(d'_\theta) - t'_\theta = U_\theta(d_\theta + x^*_\theta(c_\theta)) - t_\theta - b^a(x^*_\theta(c_\theta)) \geq 0 \tag{12}$$

where the inequality is due to IR_θ under the original contract c_θ. Next denote $V_\theta(d) = \max_{x\in\mathcal{X}} U_\theta(d+x) - b^a(x)$. We show IC constraint for type θ under c'_θ holds:

$$\begin{aligned}
U_\theta(d'_\theta) - t'_\theta &= V_\theta(d_\theta) - t_\theta && (13)\\
&\geq V_\theta(d_{\overline{\theta}}) - t_{\overline{\theta}} && (\text{due to } IC_\theta \text{ under } c)\\
&\geq U_\theta(d_{\overline{\theta}} + x_{\overline{\theta}}) - t_{\overline{\theta}} - b^a(x_{\overline{\theta}}) && (14)\\
&= U_\theta(d'_{\overline{\theta}}) - t'_{\overline{\theta}} && (15)
\end{aligned}$$

where $\overline{\theta} \in \Theta \backslash \{\theta\}$. The inequality in (14) holds because $x_{\overline{\theta}}$ is clearly not the optimal solution for $\max_{x\in\mathcal{X}} U_\theta(d_{\overline{\theta}} + x) - b^a(x)$. The last equality is directly follows how we define $c'_{\overline{\theta}}$. Due to Condition (C1), the new contract c'_θ provides more surplus to the principle compared to the original contract c_θ for all $\theta \in \Theta$. Note that the above construction is using rich contract space, Condition (C3), to design individual contract c'_θ for each type; we require individual contracts because even if the optimal contract with battery pools some types, the substitution decisions x^*_θ of those types may be different which results in separating contracts under out construction for without battery case. This completes the proof that the principal's surplus can not increase with substitution. To see cases where the principal surplus decreases see Example 1 Cases 1, 3, 4, and 5. One case to see no difference to principal's surplus is the limit case with very expensive battery cost, which makes them effectively unavailable. □

PROPOSITION 4 **(Second Best Agents' Surplus)**. *Under the conditions of Proposition 2, total agents' surplus may either increase or decrease. Moreover, this effect is heterogeneous across agents, while some types increase in surplus some others may loose or remain the same.*

Proof. See Example 1, Cases 3 to 5, in the Appendix A for different cases in the proposition. □

Proposition 4 shows that substitution technology may help or hurt the agents. Furthermore, this effect is heterogeneous across the agents while some gain and others may lose.

We define the social welfare here as the sum of principal's and agents' surplus. Proposition 3 shows the principal's surplus always decreases with substitution device available to agents, while Proposition 4 shows the agents' surplus can either increase or decrease. Therefore the social welfare always decreases if agents' surplus decreases. However, if agents' surplus increases, the social welfare can increase or decrease depending on whether increase in agents' surplus is dominated by decrease in principal's.

PROPOSITION 5 **(Second Best Social Welfare)**. *Under the conditions of Proposition 2, social welfare may increase or decrease.*

Proof. See Example 1 in Appendix A. In Case 4 agents' surplus goes down, the social welfare will also necessarily go down. In Case 3 and Case 5 the agents surplus goes up but the social welfare goes up or down depending if the increase in agents' surplus dominates reduction in principal's surplus. □

The results in Proposition 4 and 5 show that the change in agents surplus and social welfare depend on the parameters of the model. Consequently, to assess the effect of substitution devices on social welfare and agents' surplus we have to find the model parameters empirically. Furthermore, such empirical assessment allows for understanding the size of loss on principle surplus.

5. Empirical Analysis: Case of PG&E

In this section we empirically study the case of PG&E which is facing growing behind-the-meter batteries in its territory. We will first present our data and the price variations that exist in there. We will then provide our structural model for households utilities including monthly elasticities and hourly cross elasticities, and estimate its parameters using a two stage approach. We will then use these results to identify the impact of behind-the-meter batteries on PG&E optimal contract, its profit and its customers' welfare.

5.1. Data

The primary data of this study consist of household-level hourly electricity consumption recorded by smart meter for August 2010 - July 2011 and November 2011 - October 2012 in PG&E. The sample households are randomly drawn from residential households in the service area of PG&E and thus are representative for the entire population. There exist in the data around 100k residential households in the first year and around 500k in the second year. Each record of the data includes a household's account number (the unique identifier for each household), end use type (basic, all-electric, or medical-need), premise type (residential detached house, apartment, residential multi-family, etc.), climate zone (PG&E specified), tariff (flat or ToU rates), zip code, and timestamp of the consumption.

Households are by default on tiered flat rate price plan, but can opt into other programs including tiered time of use rate (ToU).[3] These are non-linear tariffs where the marginal price for electricity depends on the level of consumption in a step function. While the tiered flat rate is independent of the time of consumption, the ToU program consists of peak and off-peak hours subject to two different tiered rates. The tiers are a function of the baseline (1st tier up to baseline, then second tier to 130% of baseline, etc). The baselines are determined as a function of climate zone and season (summer/winter), as well as households' enrollment into one of the basic, all-electric, or medical-need end use types by PG&E based on their type of heating system and medical condition with default on basic. Finally, households can also choose to opt in the California Alternate Rates for Energy (CARE) program if they are eligible (based on household income and population). 25 to 30% of the households are in this program. The program provides discounted electricity rates (Shown in Table 6). [4] Table 1 shows the number of households in each specified group. Appendix B shows details of the pricing and the demographics and average consumption of households in different climate zones.

Table 1 Number of households for different rate schedules, discount programs, and end use types.

# of users	E1/E7	CARE/FERA/normal	Basic/all-electric/medical[a]
Year 1	103476/899	29320/710/75997	98898/4625/2485
Year 2	459303/2274	126344/3040/339648	421243/35153/12577

[a] There exists a very small number of mixed multi-unit households which are not listed here.

The raw data do not include information on electricity rates, households' demographic characteristics, or weather variables. We construct the load serving entity (LSE)'s rate schedules for each

[3] The opt in to the ToU program was already closed at the time of our observation (closed on January 1, 2008).

[4] Our data excludes households on another Family Electric Rate Assistance Program (FERA) which account for less than 1% of the households we observe.

billing period using PG&E's official documents.[5] We match households' 5-digit zip code with US Census 2010 data to collect income and household size. We also match each household to time series of weather temperature from Weather Underground.[6]

5.2. Price Variation

In this paper, we focus on residential tiered rate users and time-of-use users.

The prices have not significantly changed over the time period of observations. This can be observed from Figure 1 showing the price per tier; also PG&E made small change to the baseline allowance once during the consumption time of our observed users (Tables 8 and Table 9 in Appendix B).

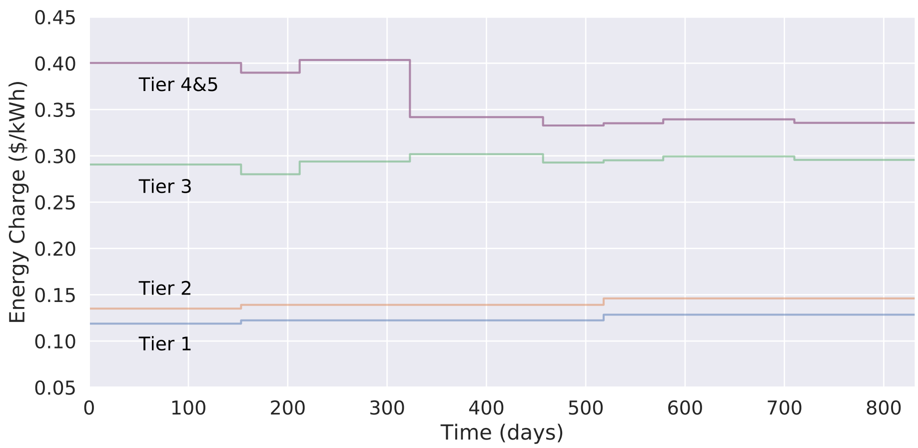

Figure 1 Residential tiered rates (E1) from August 2010 to October 2012 for PG&E. Time is measured in the number of days passing since August 1, 2010.

Also households do not respond to marginal price variation at the change of tiers as shown in Figure 2.[7] This figure shows the histograms of household monthly electricity consumption in summer relative to the baseline for both tiered users and ToU users.

We therefore leverage two other sources of price variations in our data. First, price is spatially different due to the baseline difference across climate zones as shown in Table 8 and 9. While the baseline difference is small in winter, there is major price change during summer which is used for our analysis.[8] The climate zone partition is drawn by the California Energy Commission mainly based on climatic characteristics[9]. The Energy Commission has no intention to engage households

[5] https://www.pge.com/tariffs/electric.shtml

[6] https://www.wunderground.com/

[7] Same observation is discussed at extent in Ito (2010).

[8] Here summer refers to May 1 through Oct 31 and winter refers to the rest of a year.

[9] https://ww2.energy.ca.gov/maps/renewable/building_climate_zones.html

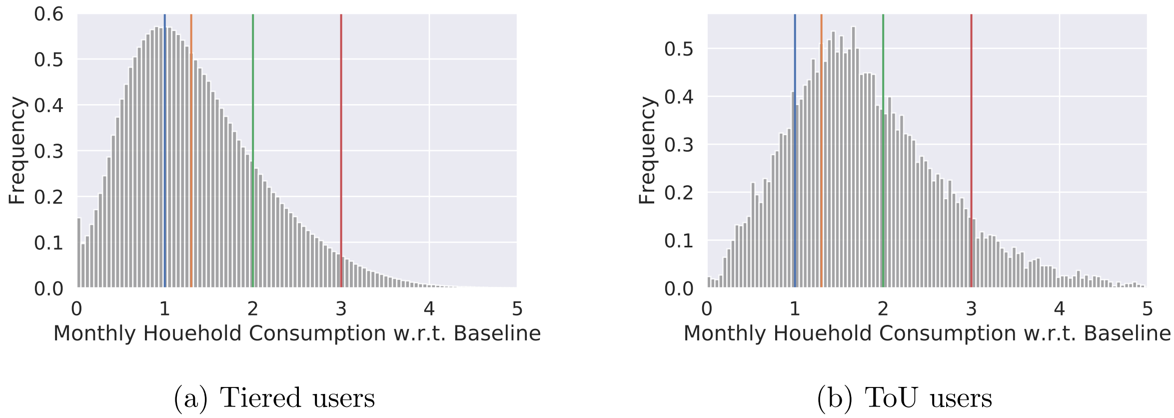

Figure 2 **Histogram of household monthly electricity consumption in summer w.r.t. the baseline. The vertical lines show the kink points of the nonlinear price schedule.**

with specific characteristics (e.g., high income) in certain climate zones. We also assume that people do not consider the amount of electricity baseline when they decide about their characteristics unobservable in our dataset that is effecting demand in locating their houses. The distribution of zip codes across climate zones is presented in Figure 3. The price variation across climate zones allows us to estimate aggregate monthly demand elasticity.

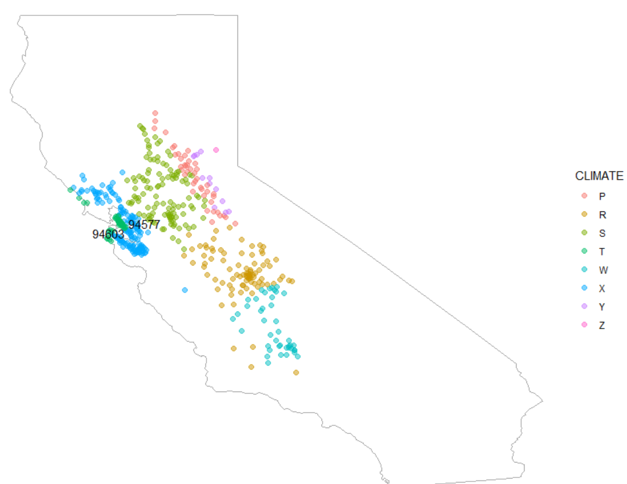

Figure 3 **Geographical distribution of zipcodes across climate zones.**

The second price variation existing in data is temporal variation for ToU users. the price is different across peak and off-peak hours for ToU tariff (see Appendix B). The price variation of peak and off peak is small in winter and therefore we use the summer data only. We discuss the details for these below. Figure 4 shows average daily demand profile of tiered and ToU users in summer. During weekdays when peak pricing exists, we observe obvious demand shift from peak to off-peak hours by ToU users as opposed to tiered users, and also as opposed to the ToU users themselves over the weekends when peak pricing is not in effect. The demand pattern we observe for ToU users give the possibility of estimating the substitution effect of consumption across different hours in a day.

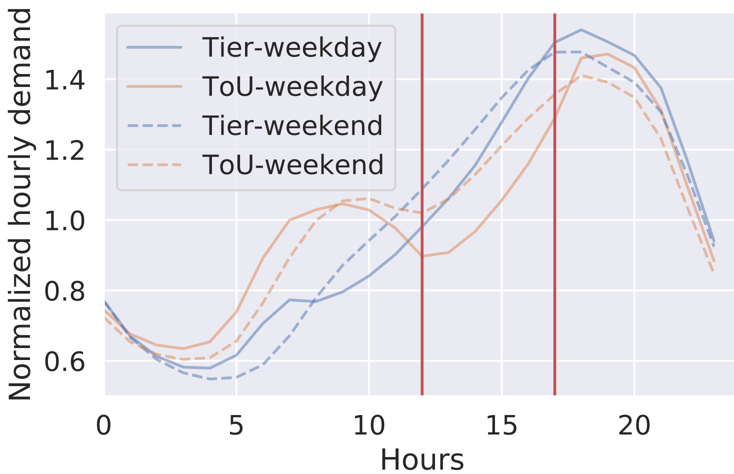

Figure 4 Average hourly demand of tiered and ToU households in summer. Each demand curve is normalized w.r.t. its own average. The time period between the two vertical lines is the peak hours if on weekdays.

5.3. Structural Model and Research Design

Finding the optimal pricing for PG&E requires estimating household's demand under counterfactual electricity prices. Note that we also model the effect of the battery on household consumption through its impact on the prices that households face. For example with battery the households can pay the price of off peak to consume at peak hours.

To estimate households electricity demand as a function of prices, we use a structural approach to estimate the household utility function $U_\theta(d)$ for consuming a vector d of electricity. We also identify type space Θ and categorize each households into its type in this space.

For household utility, we use the following function which is a combination of price elastic monthly consumption and Almost Ideal Demand System for substitution across the hours:

$$U_\theta(d) = A_\theta \exp\left(u^{AIDS}(d)\right)^{B_\theta} + C_\theta, \tag{16}$$

$$U_\theta(0) = 0 \tag{17}$$

In this model, A_θ, B_θ, C_θ are all parameters of the utility function $U_\theta(d)$, and $u_{AIDS}(d)$ represent the direct utility function from an Almost Ideal Demand System (AIDS). We remind that the AIDS model indirect utility function is:

$$v^{AIDS}(p,I)=\prod_k p_k^{-\beta_k^{AIDS}}\left(\log I-\sum_k \alpha_k^{AIDS}\log p_k-\frac{1}{2}\sum_k\sum_j \gamma_{kj}\log p_k \log p_j\right) \tag{18}$$

where p is the price vector, I is the budgeted amount, and β^{AIDS}, α_k^{AIDS}, γ_{kj} are all parameters estimated from the AIDS model below:

$$w_{ih}=\alpha_h^{AIDS}+\sum_{j=1}^{n}\gamma_{hj}\log p_{ij}+\beta_h^{AIDS}\ln\frac{I_i}{P_i}. \tag{19}$$

Here w_{ih} is the budget share of electricity in the ith household for the hth hour, p_{ij} is the price of electricity in the ith household for the jth hour, n is the number of time periods, I_i is the total electricity expenditure of household i, P_i is the electricity price index of household i, and $(\alpha^{AIDS},\gamma,\beta^{AIDS})$ are the model parameters to estimate. The matrix γ shows the consumption pattern across hours conditional on the given total expenditure for electricity. The model parameters are assumed to satisfy the following properties (Adding up, Homogeneity, and Symmetry):

$$\sum_h \alpha_h^{AIDS}=1;\ \sum_h \beta_h^{AIDS}=0;\ \sum_h \gamma_{hj}=0 \quad \forall j \tag{20}$$

$$\sum_j \gamma_{hj}=0 \quad \forall h \tag{21}$$

$$\gamma_{hj}=\gamma_{jh} \quad \forall h,j \tag{22}$$

This model is based on two-stage budgeting process on the part of consumers which has been widely adopted for energy consumption Filippini (1995b,a) and other sectors Chaudhuri et al. (2006). A two-stage budgeting process means that in the first stage the consumers decide their aggregate budget spent on a category and in the second stage they decide how to allocate the demand across different goods in that category.

We estimate the model parameters in two stages.

1. In the first stage, we estimate A_θ, β_θ, and C_θ by measuring monthly consumption response to prices using climate zones price variation in flat tiered users. We focus on flat tiered price households because there is not price difference across the hours and hence, we can find a direct utility function for AIDS, which is also independent of the AIDS parameters. This in turn allows estimating A_θ, β_θ, and C_θ.

2. In the second stage, we estimate AIDS parameters in u_{AIDS} by measuring hourly cross elasticises using price variation across peak and off-peak hours in time-of-use (ToU) households.

In the above two stage estimation, we make the following assumption about the data.

Assumption 1. *In the second stage, we assume that all consumers, both ToU tariffs and flat tiered tariffs, have utilities that can be represented with the same parametric form.*

For identifying the space of households types Θ and assigning houses to types in that space, we note that our observed households characteristics are not rich enough for addressing heterogeneity in demand elasticity. Major missing characteristics include type of appliances or lifestyle (day/night) (Dubin and McFadden 1984, Kwac et al. 2016). We use an unsupervised machine learning approach to identify these unobservable characteristics.

5.4. Households Type Identification

We enrich our observable characteristics of households including household size, income level, premise type, end-use type, weather, and other temporal variables. with a set of unobservable characteristics representing existing appliances and lifestyle. We use unsupervised machine learning to capture those. we cluster the daily load profiles of the households into nine clusters using k-means algorithm. The cluster centroids, as shown in Figure 5, are good representatives of households' life pattern especially for the time of peak energy consumption. Each household is then assigned to one centroid based on which centroid the household has the most daily profiles in. In this way, we divide the entire population into nine groups denoted by G_1 to G_9[10] based on their daily consumption patterns. The number of users in each group is represented in Figure 5. Most households belong to Cluster 3 and 9. Cluster 9 has its peak consumption at around 8 pm and probably represents a general commuting worker. Cluster 3 could represent no one living in the house, people who care little about saving energy, or people staying at home all day long.

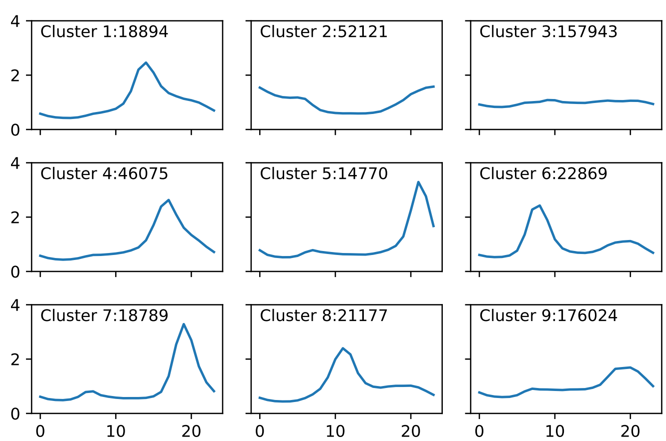

Figure 5 Daily demand profile centroids of the obtained nine clusters normalized by the average hourly demand of the day (The number of users in each cluster is also given).

[10] We decide to take nine groups to keep the number of households in each group above 10k which is a reasonable number for obtaining consistent estimates of elasticities within each group.

5.5. Estimation Stage 1: Monthly Aggregate Consumption in Response to nonlinear pricing

In the first stage we estimate parameters A_θ, B_θ, and C_θ. To isolate these parameters from those of the u_{AIDS}, we focus on aggregate monthly electricity consumption for flat tiered users; since the price is the same across hours, AIDS utility function can be calculated as follows. Set

$$\overline{d} := \sum_{h \in 24Hours} d_h \tag{23}$$

By plugging (20), (21), and (22) into (18) plus the flat rate assumption, the AIDS indirect utility function becomes

$$v_{AIDS}(p, I) = (\log I - \log p_1) = \log \overline{d} \tag{24}$$

And the household utility function is now

$$U_\theta(\overline{d}) = C_\theta + A_\theta \overline{d}^{B_\theta} \tag{flat rate}$$

The household's utility maximization problem facing flat tiered prices will be

$$\max_{\overline{d}} A_\theta \left(\overline{d}\right)^{B_\theta} - p_1 \overline{d} \tag{25}$$

The first order conditions for household demand then yield:

$$A_\theta B_\theta \overline{d}^{B_\theta - 1} - p_1 = 0 \tag{26}$$

$$\ln \overline{d} = \frac{1}{B_\theta - 1} \ln p_1 - \frac{\ln(A_\theta B_\theta)}{B_\theta - 1} \tag{27}$$

Once we have the estimated price elasticity, i.e. $\frac{1}{B_\theta - 1}$, then B_θ is determined. Next, plugin in B_θ and $\overline{d}$ into (27) the value of A_θ is determined for each household. Thus, B_θ along with the observed demand data, uniquely determines A_θ.

The basic idea of our research design is to examine electricity consumption in relation to changes in nonlinear electricity rates using households of different climate zones. Households have the same baseline within a climate zone but different baselines across climate zones. The difference in baselines leads to price variation we use in this study. Since the baseline differences are small in winter, only the data in summer is used for our analysis[11].

We present three set of regressions: (i) All-household regression, (ii) Single-border regression, (iii) Multi-border regression, which we will explain their details. To capture the heterogeneous elasticities (especially adding clusterings of Section 5.4), we use regression on all the users in our

[11] Here summer refers to May 1 through Oct 31 and winter refers to the rest of a year.

database. However, to check the results, especially against potential simultaneity issues, we also provide border regression results. In using border regression, we assume users at zip codes on two sides of the same border have similar distribution of characteristics other than their electricity tariff.[12] Note that border regression is limited in number of households to those that are in the adjacent zip codes on two sides of the borders. Therefore, border regression does not have the power to estimate heterogeneous elasticities. There are seven climate zone borders in our data: 1) S and X, 2) P and Y, 3) T and X, 4) P and S, 5) R and W, 6) R and S, 7) R and P. The number of households in each border is shown in Table 2. The regression results are shown in Table 3 and discussed at the end of this section.

Table 2 **Number of households within different borders.**

Border	None	SX	PY	TX	PS	RW	RS	RP
# households	344760	19860	6322	79611	11398	5139	1482	704

The type of household i of type θ consists of both observable characteristics of the households in the data which are climate zone/price tariff, membership in CARE program, premise type, temperature, income, and household size, and unobservable characteristic which is their lifestyle cluster identified in Section 5.4. We use the following assumption for identifying the elasticities.

Assumption 2. *The elasticity of household i with type θ is only a function of its lifestyle cluster θ and independent of the other characteristics.*

Based on Assumption 2, we run the following regression for case (i) which includes all-households.

$$\ln d_{im} = \sum_{\theta} 1\{i \in G_\theta\}\beta_{1\theta} \ln p_{im} + \beta_2 CARE_i + \sum_{t\in Month} \beta_{3t} 1(m=t) + \beta_4 PremiseType_i + \beta_5 \ln Temp_{im} + \beta_6 \ln Income_i + \beta_7 HouseholdSize_i + \eta_{im}. \tag{28}$$

where d_{im} denotes household i's average daily electricity consumption during billing month m, p_{im} is the average price of electricity of household i in month m.[13], G_θ is the group of households in lifestyle (cluster) θ. For both (ii) Single-border regression and (iii) Multi-border regression, we do not estimate cluster-varied elasticities but one elasticity for all households due to limited

[12] This particularly means households do not select the house at purchase time based on electricity tariff

[13] There are debates on whether users respond to marginal price or average price (Borenstein 2009, Ito 2010). The spatial price discontinuity in one retailer territory in our data, unlike two territories as in Ito (2010), does not allow us to distinguish between the marginal price and average price because the positive correlation between the two prices always exist. We thus follow Ito (2010) to use average price as the price variable because our work also uses California's household consumption data.

household number as mentioned previously.[14] In running the above regression we have the following assumption about time invariance of household elasticities.

Assumption 3. *The elasticity is constant over time.*

The above assumption is based on the fact that data collection period is short enough compared to long time elasticity changes due to new technologies or economical shocks.

There are both simultaneity and endogeneity in the regression above. Simultaneity is because higher temperature climate zones, which naturally have higher demand in summer, are assigned higher baseline (lower price) by PG&E. To address this simultaneity, we include the temperature variable in the regression. As mentioned, to check that our technique properly addresses this simultaneity, we also test our results with border analysis.

There exists endogeneity of demand on prices in regressions (i), (ii), and (iii) because the price is a function of demand through the non-linear tiered tariff. To address the endogeneity issue, we need an instrument variable (IV) that is correlated with the price (relevance) but is uncorrelated with the demand (exclusion restriction) conditional on the observable. We first consider the IV for regression (i). The baseline b_i is a natural choice of IV because price is a function of the baseline and baselines differ for climate zones. Specifically, the relevance condition of IV b_i is that $\psi_1 \neq 0$ for

$$\begin{aligned} \ln p_{im} =& \psi_1 b_i + \delta_2 CARE_i + \sum_{t \in Month} \delta_{3t} 1(m=t) + \delta_4 PremiseType_i \\ &+ \delta_5 \ln Temp_{im} + \delta_6 \ln Income_i + \delta_7 HouseholdSize_i + r_{im}. \end{aligned} \tag{29}$$

Note that for succinctness, we omit the lifestyle dummy $1\{i \in G_\theta\}$ on both sides of the above equation. In Table 3 the results of regressions with the simple baseline IV b_i (marked as (i/iii)-base IV) are order of magnitude above the literature. This suggests we might need an alternative IV.

We propose an alternative IV $b_i * \sum_n Temp\_round_n$ which is interaction of baseline and rounded temperature dummy and marked as (i/iii)-Alt IV in Table 3. The rounded temperature $Temp\_round_n$ is a dummy variable which is 1 if $Temp_{im} \in [n, n+1)$ and 0 otherwise. For example, all temperature in $[59, 60)$ F will belong to $Temp\_round_{59}$.

Note that both the base IV and the alternative IV have high F-statistic in the first-stage regression (both larger than 10^4), indicating that they are not week IVs. However, the alternative IV is

[14] Also, for single border regression, the temperature variable is in its original value (not the logarithm) because the temperature variation on one border is small and can therefore be captured by a linear relationship. Also we do not include the income variable and the household size variable in the regression. The reason is that our data for these two variables is at an aggregate level and does not reflect the true distribution of these characteristics. Therefore, by excluding the income and the household size variables, we are assuming the distribution of these two variables are the same across the climate zone border.

required because based on the marginal price in each tier (E1 as shown in Table 10), the value of ψ_1 in (29) is dependent on which tier the users are in. For example, Tier 2 price and Tier 1 price are closer than Tier 2 price and Tier 3 price. The price difference among Tier 3, 4 and 5 is also smaller than between Tier 2 and Tier 3. That said, households with demand near the threshold of Tier 1 and 2 or Tier 3, 4 and 5 will have lower ψ_1 in magnitude than households with demand near the threshold of Tier 2 and 3. Therefore, if we pool all users together and obtain only one ψ_1, the linear regression adopts L2 loss and ψ_1 will be biased towards a lower magnitude than it should be, which means biased towards Tiers 3,4, and 5 whose prices are higher. A lower magnitude of ψ_1 will cause lower variation of $\ln \hat{p}_{im}$ and thus higher magnitude of estimated price elasticity than it should be. This is exactly what we observe in Table 3 for base IV. These observations suggest that the value of ψ_1 is conditioned on the household demand level or the temperature level, since simultaneity between demand and temperature exists, and that is why the alternative IV is used. The relevance condition of the alternative IV is that $\psi_{1n} \neq 0$ $(n = 1, ..., N)$ for

$$\ln p_{im} = \sum_n \psi_{1n} b_i * Temp\_round_n + \delta_2 CARE_i + \sum_{t \in Month} \delta_{3t} 1(m = t) + \delta_4 PremiseType_i + \delta_5 \ln Temp_{im} + \delta_6 \ln Income_i + \delta_7 HouseholdSize_i + r_{im}. \tag{30}$$

Note that the interaction $b_i * Temp\_round_n$ does not contain separate b_i or $Temp\_round_n$ terms and therefore it has no multicollinearity problem with existing $\ln Temp_{im}$ term in the regression, nor does it affect the discussion of exclusion restriction in Assumption 4 below.

For regression (iii) which is based on households on all borders, we assume households on two sides of the same border have almost similar temperature. Therefore, as a special case of regression (i), we instrument on the interaction of baseline b_i and border categorical variable $Border_i$ (the border to which household i belongs) for addressing the endogeneity in regression (iii). We no longer need to interact the lifestyle dummy as we are not solving for heterogeneous price elasticity based on lifestyle clusters in regression (iii). For regression (ii) which includes households on a single border, we see it as a special case of regression (iii) and instrument on the baseline b_i. In other words, we do not need to interact the border categorical variable $Border_i$ because there is only one border in this regression.

In using the above IVs for the regressions we have the following independence assumption.

Assumption 4. *Households do not choose their house based on temperature preferences, and do not choose their appliances based on electricity prices.*

Regression results- The regression coefficients of (i),(ii), and (iii) are shown in Table 3 and Table 4. Regression (ii) and (iii) serve as a check for the results obtained in regression (i) especially whether simultaneity issue is addressed in (i), because there is less temperature variation close to the same border which eliminates simultaneity issues for regressions (ii) and (iii). For all regressions, we have reported both without IV and with IV results to show how IV corrects the endogenity issue. We run regression (ii) for SX border. The reason for choosing SX border is that among all the borders, TX is the most populated one and SX is second. However, the number of households on the two sides of TX border is very unbalanced. In particular, the number of households on the "T" side is less than the number of households on each side of the SX border. The SX border has balanced number of households on the two sides.

Based on the results in Table 3 and Table 4, for regressions without IV, we observe price elasticities estimated to be positive. This suggests that the regressions suffer from the endogeneity issue. When including IVs, the price elasticities for the nine clusters range between -0.323 and -0.649 and are all significant. These elasticities are of comparable amount to those reported in literature such as in Ito (2014), Deryugina et al. (2020), Neenan and Eom (2008). Moreover, both price elasticities from regression (ii)-SX and (iii) fall into the elasticity range estimated from regression (i). This indicates that our all-household regression results are acceptable and may not suffer from issues such as simultaneity. Furthermore, comparing regression (ii)-SX with regression (iii), their price elasticities are within the range of standard deviation of each other and thus can cross validate.

We have the following observations from the regressions. First, households having peak consumption during the day such as Clusters 1, 4, 6, and 8 tend to have higher price elasticity. Cluster 9 has its peak consumption at around 8 pm and probably represents a general commuting worker. They also have high price elasticity. Second, households having peak consumption in late evening such as Cluster 2 and 5 tend to have lower price elasticity. The intuition might be those people who arrive home late have less ability to cut their consumption in response to price. Third, Cluster 3 has the second lowest price elasticity, which aligns with our inference from its load profile; Cluster 3 represents no one living in the house, people who care little about saving energy, or people staying at home all day long. Also, the other coefficients show that demand is lower for CARE users, but higher for high temperature areas, high-income areas, large household size. The reason is that high income, and large household sizes are have more electricity appliances and thus higher demand; CARE is the low income program and therefore lower consumption of electricity. Finally high temperature is positively correlated with high air conditioning use.

Estimation of A_θ and B_θ- We use the results of (i) All Household regression with IV in (28) to estimate A_θ and B_θ. From (27) and (28) A_θ and B_θ (varied by cluster type) can be calculated by

$$B_\theta = \frac{1}{\beta_{1\theta}} + 1 \tag{31}$$

Table 3 **Regression coefficients for all regressions (Premise type omitted)**

Regression	(i), no IV	(i)-Alt IV	(i)-base IV	(ii)-for SX boarder	(iii)-Alt IV	(iii)-base IV
$\ln p_{im}$	-	-	-	-0.333	-0.425	-1.865
	-	-	-	(0.081)***	(0.029)***	(0.047)***
$\ln p_{im} * cluster_1$	2.222	-0.649	-2.359	-	-	-
	(0.002)***	(0.015)***	(0.024)***	-	-	-
$\ln p_{im} * cluster_2$	2.383	-0.323	-1.933	-	-	-
	(0.002)***	(0.014)***	(0.023)***	-	-	-
$\ln p_{im} * cluster_3$	2.333	-0.412	-2.047	-	-	-
	(0.002)***	(0.015)***	(0.023)***	-	-	-
$\ln p_{im} * cluster_4$	2.218	-0.601	-2.287	-	-	-
	(0.002)***	(0.015)***	(0.024)***	-	-	-
$\ln p_{im} * cluster_5$	2.318	-0.388	-2.004	-	-	-
	(0.002)***	(0.014)***	(0.023)***	-	-	-
$\ln p_{im} * cluster_6$	2.278	-0.499	-2.156	-	-	-
	(0.002)***	(0.015)***	(0.024)***	-	-	-
$\ln p_{im} * cluster_7$	2.290	-0.424	-2.045	-	-	-
	(0.002)***	(0.014)***	(0.023)***	-	-	-
$\ln p_{im} * cluster_8$	2.267	-0.580	-2.273	-	-	-
	(0.002)***	(0.015)***	(0.024)***	-	-	-
$\ln p_{im} * cluster_9$	2.248	-0.556	-2.231	-	-	-
	(0.002)***	(0.015)***	(0.024)***	-	-	-
$CARE_i$	1.122	-0.268	-1.097	-0.282	-0.213	-0.908
	(0.001)***	(0.007)***	(0.012)***	(0.043)***	(0.014)***	(0.023)***
$Temp_{im}$	-	-	-	0.104	-	-
	-	-	-	(0.004)***	-	-
$\ln Temp_{im}$	3.045	4.372	5.159	-	4.240	5.204
	(0.006)***	(0.011)***	(0.015)***	-	(0.028)***	(0.041)***
$\ln Income_i$	0.132	0.355	0.487	-	0.407	0.576
	(0.001)***	(0.002)***	(0.003)***	-	(0.005)***	(0.007)***
$HouseholdSize_i$	0.103	0.104	0.104	-	0.095	0.077
	(0.001)***	(0.001)***	(0.001)***	-	(0.003)***	(0.003)***
Sep_m	-0.052	-0.244	-0.358	-0.497	-0.260	-0.354
	(0.001)***	(0.002)***	(0.003)***	(0.010)***	(0.004)***	(0.005)***
Oct_m	0.229	0.072	-0.022	0.174	0.115	0.104
	(0.001)***	(0.002)***	(0.003)***	(0.026)***	(0.004)***	(0.004)***
May_m	0.396	0.236	0.140	0.479	0.290	0.292
	(0.002)***	(0.002)***	(0.003)***	(0.045)***	(0.003)***	(0.006)***
Jun_m	0.147	0.084	0.046	0.172	0.110	0.115
	(0.001)***	(0.002)***	(0.002)***	(0.015)***	(0.003)***	(0.004)***
Jul_m	-0.077	-0.200	-0.274	-0.276	-0.187	-0.250
	(0.001)***	(0.002)***	(0.002)***	(0.009)***	(0.004)***	(0.005)***

$$-\frac{\ln(A_\theta B_\theta)}{B_\theta - 1} = \beta_2 CARE_i + \sum_{t \in Month} \beta_{3t} 1(m = t) + \beta_4 PremiseType_i + \beta_5 \ln Temp_{im} + \beta_6 \ln Income_i + \beta_7 HouseholdSize_i + \eta_{im} \tag{32}$$

Here B_θ is only dependent on the price elasticity of type θ which by Assumption 2 only depends on the lifestyle cluster G_θ household i is in. Thus for the nine lifestyle clusters, we have nine values

Table 4 **Regression coefficients for all regressions -continued (Premise type omitted)**

Regression	(ii)-for SX boarder, no IV	(iii)-no IV
$\ln p_{im}$	2.237	2.377
	(0.247)***	(0.004)***
$\ln p_{im} * cluster_1$	-	-
	-	-
$\ln p_{im} * cluster_2$	-	-
	-	-
$\ln p_{im} * cluster_3$	-	-
	-	-
$\ln p_{im} * cluster_4$	-	-
	-	-
$\ln p_{im} * cluster_5$	-	-
	-	-
$\ln p_{im} * cluster_6$	-	-
	-	-
$\ln p_{im} * cluster_7$	-	-
	-	-
$\ln p_{im} * cluster_8$	-	-
	-	-
$\ln p_{im} * cluster_9$	-	-
	-	-
$CARE_i$	1.067	1.141
	(0.005)***	(0.003)***
$Temp_{im}$	0.196	-
	(0.002)***	-
$\ln Temp_{im}$	-	2.363
	-	(0.016)***
$\ln Income_i$	-	0.077
	-	(0.002)***
$HouseholdSize_i$	-	0.130
	-	(0.002)***
Sep_m	-0.492	-0.078
	(0.007)***	(0.003)***
Oct_m	0.833	0.137
	(0.010)***	(0.003)***
May_m	1.642	0.285
	(0.019)***	(0.003)***
Jun_m	0.500	0.098
	(0.007)***	(0.003)***
Jul_m	-0.301	-0.063
	(0.006)***	(0.003)***

of B_θ :

$$[-0.5408, -2.0960, -1.4272, -0.6639, -1.5773, -1.0040, -1.3585, -0.7241, -0.7986] \tag{33}$$

A_θ however also depends on the other characteristics of the household including CARE program $CARE_i$, the relevant month $1(m = t)$, premise type $PremiseType_i$, temperature $\ln Temp_{im}$, income

$\ln Income_i$, and household size $HouseholdSize_i$. The weighted sum of these characteristics determines the A_θ. We calculate A_θ for two weighted sums here that are of high representation in data, $-\frac{\ln(A_\theta B_\theta)}{B_\theta - 1}$ equal 1.86 and equal 2.1. A higher value of $-\frac{\ln(A_\theta B_\theta)}{B_\theta - 1}$ is correlated with higher demand, higher income, larger household size, being in higher temperature zones, and not in CARE program. This could be the effect of one such variable or confluence of them. $-\frac{\ln(A_\theta B_\theta)}{B_\theta - 1} = 2.1$ is among the most prevalent values in the total population and $-\frac{\ln(A_\theta B_\theta)}{B_\theta - 1} = 1.86$ is more seen in the less price responsive population. The values of A_θ depends on the B_θ which can take nine different values depending on lifestyle cluster k_θ as discussed above. For a representative household with weighted sum of the characteristics equal 1.86 or 2.1, i.e. $-\frac{\ln(A_\theta B_\theta)}{B_\theta - 1} = 1.86$ or $-\frac{\ln(A_\theta B_\theta)}{B_\theta - 1} = 2.1$, the 9 values of A_θ are shown in Table 5.

Table 5 A_θ **for different types of households.**

$-\frac{\ln(A_\theta B_\theta)}{B_\theta - 1}$	Type 1	Type 2	Type 3	Type 4	Type 5	Type 6	Type 7	Type 8	Type 9
1.86	-32.48	-151.18	-64.00	-33.26	-76.56	-41.41	-59.17	-34.11	-35.53
2.1	-47.01	-317.83	-114.59	-49.59	-142.12	-66.98	-104.21	-51.60	-54.70

5.6. Stage 2: Hourly Cross Elasticities in Response to Time-of-Use (ToU) pricing

In this section we estimate the price cross elasticities across 24 hours. To do so we use peak/off peak hour price variations for the ToU users. To estimate u_{AIDS} parameters, we use AIDS standard linear approximation equations (LA-AIDS) which only give the indirect utility function.

Following Filippini (1995a), because of only two time periods of prices in the ToU pricing, only a 2×2 price elasticity matrix between peak and off-peak hours can be estimated with the data. Extending this cross elasticity to more granular matrix requires additional structural assumptions. Particularly estimating a 24×24 elasticity matrix can achieved in two steps. First since the smart meter data we observe is at hourly level, even with only peak/off-peak price, we can estimate how demand in each hour changes with respect to peak/off-peak price change, that is, price elasticity for dimension of up to 24×2. Second, by introducing structures in the elasticity matrix, we can estimate the price elasticity matrix with a more granular time-scale, 24×24. We discuss the details below.

For the AIDs model in (19), we approximate the price index P_i with linear AIDS (LA-AIDS) model in Buse (1994) by

$$\ln P_i = \sum_{j=1}^{n} w_{ij} \ln p_{ij} \tag{34}$$

We first estimate a 2×2 cross elasticity matrix between peak and off-peak hours by using γ_{hP} (and γ_{hO}) that are sum of all peak (off-peak respectively) hours in the hth row of γ. We will later discuss how we extend these to full γ using structural assumptions. Finally, we use the following regression which incorporates the observable characteristics of demographics and temperature effect:

$$
\begin{aligned}
w_{ih} &= \gamma_{hP} \ln p_{hP} + \gamma_{hO} \ln p_{hO} + \beta_h^{AIDS} \ln \frac{I_i}{P_i} + \sum_{t \in Month} \alpha_{h1t}^{AIDS} Time_t + \alpha_{h2}^{AIDS} PremiseType_i + \eta_{ih} \\
&= \gamma_{hP} \ln \frac{p_{hP}}{p_{hO}} + \beta_h^{AIDS} \ln \frac{I_i}{P_i} + \sum_{t \in Month} \alpha_{h1t}^{AIDS} Time_t + \alpha_{h2}^{AIDS} PremiseType_i + \eta_{ih} \qquad (35)
\end{aligned}
$$

REMARK 9. Note that since ToU users are much smaller in number in our data than flat tiered users, we do not include any zip code related control in the regression, or any clustering heterogeneity for lack of identification power. Therefore our estimates for the AIDS model should be interpreted as average across all clusters and all zip codes.

There exists endogeneity because from the non-linear pricing, the shock to peak/off-peak price ratio η_{ih} is a function of demand and hence a function of w_{ih}. To address the endogeneity, we use jointly the baseline b_i and the low-income tariff program indicator $CARE_i$ as instruments. To be valid instruments, b_i[15] and $CARE_i$ should be correlated with $\frac{p_{hP}}{p_{hO}}$ yet uncorrelated with η_{ih} condition on observables. The reason we use CARE in addition to baseline as IV is that the proportion of CARE ToU users in different climate zones varies significantly due to the small number of ToU users we observe. We already provided a discussion on baseline as IV in Section 5.5. CARE is correlated with $\frac{p_{hP}}{p_{hO}}$ through the price discount. We note that since people should opt in for the CARE program, there may be endogeneity between the CARE IV and demand shock condition on observbles. We assume such endogeneity does not exists.

ASSUMPTION 5. *Self-selection into the CARE program is independent of the demand shock conditioned on the observable variables.*

We estimate the values of γ_{hP}, a 24-dimension vector and the values of γ_{hO} are just the opposite, as discussed earlier

$$
\gamma_P = [0.012, 0.009, 0.009, 0.012, 0.012, 0.014, 0.011, 0.006, 0.010, 0.011, 0.018, 0.017, \qquad (36)
$$

$$
0.004, -0.023, -0.054, -0.072, -0.066, -0.044, -0.007, 0.014, 0.034, 0.035, 0.022, 0.015] \qquad (37)
$$

where all of the 24 estimated values are of statistical significance with p-value less than 0.05. For h being peak hours, γ_{hP} is negative, indicating that the electricity budget share of hour h in peak hours will be lower if the ratio of peak and off-peak prices is high. This aligns with intuition. Comparing to existing work, Filippini (1995a) have estimated $\sum_{h \in \text{peak}} \gamma_{hP}$ to be -0.263, which in our case is -0.267. Such a comparison validates our estimated values.

[15] Note that the alternative IV of the interaction of baseline and rounded temperature is no longer used as the ratio of peak and off-peak prices, $\frac{p_{hP}}{p_{hO}}$, weakens the difference across tiers.

After estimating γ_{hP} and γ_{hO}, we construct the 24×24 γ matrix leveraging the three conditions imposed by AIDS (20), (21), and (22) and that

$$\gamma_{hP} = \sum_{j=13}^{18} \gamma_{hj} \quad \forall i, \tag{38}$$

$$\gamma_{hO} = \sum_{j=1}^{12} \gamma_{hj} + \sum_{j=19}^{24} \gamma_{hj} \quad \forall i, \tag{39}$$

$$\gamma_{h,h-1} = \gamma_{h,h+1} \quad \forall i, \tag{40}$$

where the first two constraints are due to consumers facing the same electricity price throughout peak hours or off-peak hours and the third constraint is for equal revenue-share price elasticities (γ matrix) of neighboring hours following Ata et al. (2018). As mentioned above, extension to 24×24 hours elasticity requires extra structural assumption since with only 2 prices with peak and off-peak hours, estimating γ still requires solving an underdetermined linear system of equations. We apply the minimum-norm solution[16] here which has the implication of keeping the 2-norm of γ small.

The second stage regression results are reported in Table 12 in Appendix C. To provide easier interpretation of the matrix γ, here we report conditional price elasticities according to Buse (1994) via

$$\theta^c_{hj} = -\delta_{hj} + \frac{\gamma_{hj}}{w_h} - \frac{\beta^{AIDS}_h}{w_h} \frac{w_j + \sum_k \gamma_{kj} \ln p_k}{1 + \sum_k \beta_k \ln p_k} \tag{41}$$

where δ_{hj} is Kronecker delta with $\delta_{hj} = 1 \forall h = j$ and $\delta_{hj} = 0 \forall h \neq j$. The unconditional price elasticities are then given by merging the results of the first stage and second stage as follows:

$$\theta^u_{hj} = \theta^c_{hj} + \frac{\partial \ln \overline{d}}{\partial \ln P} \frac{\partial P}{\partial \ln p_j} = \theta^c_{hj} + \beta_\theta w_j \tag{42}$$

Figure 6 shows the results of the estimated unconditional price elasticities. First, we observe that all own price elasticity (the diagonal elements of the matrix) are negative, with peak hours being the highest (in absolute value), which shows households are more sensitive to price changes when the price is high. We also find that cross price elasticity between a peak hour and an off-peak hour are mostly positive (substitutes) yet cross price elasticity between two hours within the same peak or off-peak period is often negative (complements). The next observation we have is that neighbor hours mostly have negative cross price elasticity (complements). This is reasonable because if one wants to change consumption in a certain hour, it is probably reducing consumption in neighbor hours due to multiple hour time cycle of the appliances. We finally note that our conditional price

[16] This is a general recipe dealing with underdetermined equations. We use it in part to avoid unreasonably large elasticities.

elasticity results are comparable with Yang et al. (2014), Filippini (1995a) which also use AIDS model to estimate cross price elasticity of electricity consumption.

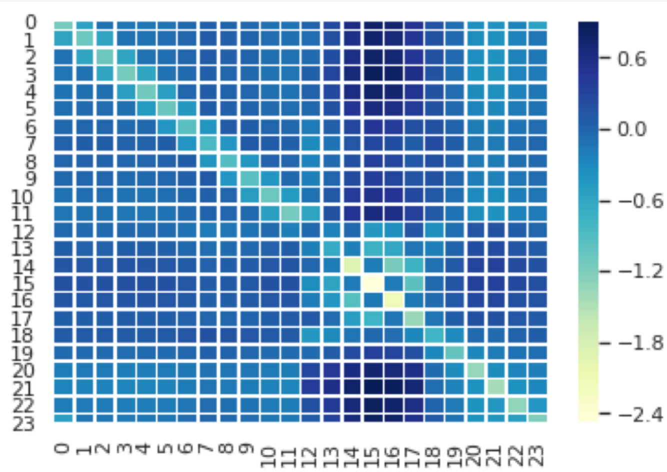

Figure 6 **Estimated unconditional price elasticities.**

5.7. Optimal PG&E Contract facing behind-the-meter Battery

In this section, we use the estimated household utilities in Section 5.4 to solve for PG&E optimal contract offered to the households when behind-the-meter batteries are accessible to the households. We compare four cases: first best without battery, first best with battery, second best without battery, and second best with battery. Here, by first best we refer to the case where the households incentive constraints are not included in the optimization. In solving these optimization problems, we assume Conditions (C1) to (C3) hold. Therefore from no arbitrage opportunity of Condition (C1), we know first best with battery is the same as first best without battery. Thus we refer to both cases as first best. We also use the zero-substitution result in Proposition 2 which allows searching for the second best optimal contract only within the space of contracts with zero substitution by households.

For second best optimal contract, PG&E should separate two types of household relevant characteristics: (a) characteristics that PG&E observes and is allowed/able to contract upon. (b) characteristics that PG&E does not observe or is not allowed to contract on but agents can change their demand based on that. We assume all the characteristics observable to PG&E are also observable to households at the time of contract selection. For each fixed values of the type (a) characteristics, PG&E offers a specific menu of contracts. The number of incentive constraints are equal to the number of type (b) characteristics. In the rest of this section we assume a household type (a) characteristic is climate zone/price tariff, $CARE$, $PremiseType$, $HouseholdSize$, $Temp$, and $Income$, and a household type (b) characteristic is lifestyle cluster G_k.

There are two computational challenges for PG&E first best and second best contracts. First, our structural utility model involves indirect utility function of AIDS while the program in Section 4.1 uses direct utility functions. We use the notion of perceived prices for addressing the indirect utility function. Second the optimizations are non-convex due to non-convex IR constraints. We use a linear approximation of the IR constraints for addressing convexity.

We present our approach for two time period consumption, peak and off-peak, for computational simplicity and ease of presentation. Here the product bundle only consists of two goods (peak and off-peak consumption). The solution for a more granular pricing, for example 24 hours, is achievable using similar approach we present here, and using more granular household utility estimation e.g. the 24 hour cross elasticity matrix we presented in Section 5.6.

5.7.1. Approach for solving the PG&E optimization **First best-** The first best problem is to maximize the principle surplus considering the IR constraints but not IC constraints. Since we only have the indirect utility function of the AIDS model, we rewrite the PG&E optimization problem considering the perceived prices $p_{\theta\theta'}$, the perceived price vector of household type θ under contract for type θ'. The formulation is as follows

$$\max_{t_\theta, d_\theta, p_{\theta\theta} | \theta \in \Theta} E_\theta \{t_\theta - c^T d_\theta\} \tag{43}$$

$$s.t. \quad V_\theta(p_{\theta\theta}, t_\theta) \geq t_\theta, \forall \theta \tag{IR}$$

$$w_{\theta\theta h} = \alpha_h^{AIDS} + \sum_{j=1}^{n} \gamma_{hj} \log p_{\theta\theta j} + \beta_h^{AIDS} \ln \frac{t_\theta}{P_{\theta\theta}}, \forall \theta, h \tag{AIDS}$$

$$w_{\theta\theta h} = d_{\theta h} p_{\theta\theta h} / t_\theta, \forall \theta, h \tag{44}$$

$$\sum_h d_{\theta h} \leq d_\theta^{max}, \forall \theta, h \tag{45}$$

Here $V_\theta(p_{\theta\theta}, t_\theta)$ is the indirect utility obtained from the AIDS model, n is the number of time periods, $p_{\theta\theta}$ is the perceived price vector of household type θ under contract for type θ, and d_θ^{max} is the maximum daily demand possible for the household. p_θ is perceived price because in the contract only total payment and consumption is provided, but the perceived price serves as a connection between the offered contract and the indirect utility function. Such a connection is characterized by the second constraint, the budget share equation of AIDS. The constraint in (45) marks a limit in real world that the total demand is limited by conditions such as number of appliances, and so the estimated utility function may only be applicable for a limited range of demand.

In the above optimization, the objective function is linear, but the IR constraints are non-convex. Thus to numerically solve the optimization, we modify the IR constraints to a linear version using following steps. First plugging in the utility function:

$$C_\theta + A_\theta \exp(V_\theta(p_{\theta\theta}, t_\theta))^{B_\theta} \geq t_\theta \tag{46}$$

Therefore,

$$B_\theta V_\theta(p_{\theta\theta}, t_\theta) \leq \log(\frac{t_\theta - C_\theta}{A_\theta}) \tag{47}$$

which implies

$$B_\theta[\log t_\theta - \sum_k \alpha_k^{AIDS} \log p_{\theta\theta k} - \frac{1}{2}\sum_k \sum_j \gamma_{kj} \log p_{\theta\theta k} \log p_{\theta\theta j}] \leq \log(\frac{t_\theta - C_\theta}{A_\theta})/[\prod_k (p_{\theta\theta k})^{-\beta_k^{AIDS}}]. \tag{48}$$

Assuming there are only two time periods and $p_1^i \geq p_2^i$, that is, peak price higher than off-peak price, the IR constraint can be relaxed to

$$B_\theta[\log t_\theta - \sum_k \alpha_k^{AIDS} \log p_{\theta\theta k} - \frac{1}{2}\gamma_{11}(\log p_{\theta\theta 1} - \log p_{\theta\theta 2})] \leq \log(\frac{t_\theta - C_\theta}{A_\theta})/[\prod_k (p_{\theta\theta k})^{-\beta_k^{AIDS}}] \tag{49}$$

where the left hand side is convex (by considering $\log p$ as the variable) and the right hand side is quasi-concave.

Second best without battery- The second best contract without battery is to maximize the principle surplus considering both IR constraints and IC constraints. We use the same techniques as for the first best to solve this optimization. The formulation is as follows

$$\max_{t_\theta, d_\theta, p_{\theta\theta'} | \theta, \theta' \in \Theta} E_\theta\{t_\theta - c^T d_\theta\} \tag{50}$$

$$s.t. \quad V_\theta(p_{\theta\theta}, t_\theta) \geq t_\theta, \forall \theta \tag{IR}$$

$$V_\theta(p_{\theta\theta}, t_\theta) - t_\theta \geq V_\theta(p_{\theta\theta'}, t_{\theta'}) - t_{\theta'}, \forall \theta, \theta' \tag{IC}$$

$$w_{\theta\theta' h} = \alpha_h^{AIDS} + \sum_{j=1}^n \gamma_{hj} \log p_{\theta\theta' j} + \beta_h^{AIDS} \ln \frac{t_{\theta'}}{P_{\theta\theta'}}, \forall \theta, \theta', h \tag{AIDS}$$

$$w_{\theta\theta' h} = d_{\theta' h} p_{\theta\theta' h} / t_{\theta'}, \forall \theta, \theta', h \tag{51}$$

$$\sum_h d_{\theta h} \leq d_\theta^{max}, \forall \theta, h \tag{52}$$

where the right hand side of the IC constraint is the surplus of user type θ when facing the contract designed for user type θ', $p_{\theta\theta'}$ is the perceived price vector of user type θ under contract for type θ'. Note that it is easy to observe under second best without battery, $p_{\theta\theta'} = p_{\theta'\theta'}$.

Second best with battery- The formulation of the second best problem is a two-stage optimization where in the first stage households solve the optimal operation problem facing a contract menu and in the second stage the retailer solves for its optimal contract menu. The first stage problem for household of type θ given a contract $(d_{\theta'}, t_{\theta'})$ is the following.

$$(x_{\theta\theta'}^*, p_{\theta\theta'}^*) \in \arg\max_{x_{\theta\theta'}, p_{\theta\theta'}} V_\theta(p_{\theta\theta'}, t_{\theta'} + b^a(x_{\theta\theta'})) - t_{\theta'} - b^a(x_{\theta\theta'}) \tag{53}$$

$$s.t. \quad w_{\theta\theta' h} = \alpha_h^{AIDS} + \sum_{j=1}^n \gamma_{hj} \log p_{\theta\theta' j} + \beta_h^{AIDS} \ln \frac{t'_\theta + b^a(x_{\theta\theta'})}{P_{\theta\theta'}}, \forall h \tag{AIDS}$$

$$w_{\theta\theta' h} = (d_{\theta' h} + x_{\theta\theta' h}) p_{\theta\theta' h} / (t_{\theta'} + b^a(x_{\theta\theta'})), \forall h \tag{54}$$

$$p_{\theta\theta' 1} \geq p_{\theta\theta' 2} \tag{55}$$

where the third constraint is to ensure the solution does not include reverse storage in time. The second stage problem is

$$\max_{(d_\theta,t_\theta)|\theta\in\Theta} E_\theta\{t_\theta - c^T d_\theta\} \tag{56}$$

$$s.t. \quad V_\theta(p^*_{\theta\theta}, t_\theta + b^a(x^*_{\theta\theta})) \geq t_\theta + b^a(x^*_{\theta\theta}),\ \forall\theta \tag{IR}$$

$$V_\theta(p^*_{\theta\theta}, t_\theta + b^a(x^*_{\theta\theta})) - t_\theta - b^a(x^*_{\theta\theta}) \geq V_\theta(p^*_{\theta\theta'}, t_{\theta'} + b^a(x^*_{\theta\theta'})) - t_{\theta'} - b^a(x^*_{\theta\theta'}), \forall\theta,\theta' \tag{IC}$$

$$\sum d_\theta \leq d^{max}_\theta,\ \forall\theta \tag{57}$$

Due to the zero-battery result in Section 4.2 we know at the optimal contract solution $x^*_{\theta\theta} = 0, \forall\theta \in \Theta$. Therefore, the second stage problem can be re-written as

$$\max_{(d_\theta,t_\theta)|\theta\in\Theta} E_\theta\{t_\theta - c^T d_\theta\} \tag{58}$$

$$s.t. \quad V_\theta(p^*_{\theta\theta}, t_\theta) \geq t_\theta,\ \forall\theta \tag{IR}$$

$$V_\theta(p^*_{\theta\theta}, t_\theta) - t_\theta \geq V_\theta(p^*_{\theta\theta'}, t_{\theta'} + b^a(x^*_{\theta\theta'})) - t_{\theta'} - b^a(x^*_{\theta\theta'}), \forall\theta,\theta' \tag{IC}$$

$$\sum d_\theta \leq d^{max}_\theta,\ \forall\theta \tag{59}$$

$$x^*_{\theta\theta} = 0,\ \forall\theta \tag{60}$$

We solve the two-stage problem by plugging in the Karush–Kuhn–Tucker (KKT) conditions of the first-stage problem for all household type θ facing all contract types $(d_{\theta'}, t_{\theta'})$ into the constraints of the second-stage problem. The KKT conditions for type θ facing contract $(d_{\theta'}, t_{\theta'})$ are

$$-\nabla[V_\theta(p^*_{\theta\theta'}, t_{\theta'} + b^a(x^*_{\theta\theta'})) - t_{\theta'} - b^a(x^*_{\theta\theta'})] + \mu_1\nabla(-x^*_{\theta\theta'}) + \mu_2\nabla(p^*_{\theta\theta'2} - p^*_{\theta\theta'1}) \tag{61}$$

$$+\sum_h \lambda_h \nabla[w_{\theta\theta'h} - (\alpha^{AIDS}_h + \sum_{j=1}^n \gamma_{hj}\log p^*_{\theta\theta'j} + \beta^{AIDS}_h \ln\frac{t'_\theta + b^a(x^*_{\theta\theta})}{P_{\theta\theta'}})] = 0, \forall h \tag{62}$$

$$w^*_{\theta\theta'h} = \alpha^{AIDS}_h + \sum_{j=1}^n \gamma_{hj}\log p^*_{\theta\theta'j} + \beta^{AIDS}_h \ln\frac{t_{\theta'} + b^a(x^*_{\theta\theta})}{P_{\theta\theta'}}, \forall h \tag{63}$$

$$x^*_{\theta\theta'} \geq 0 \tag{64}$$

$$p^*_{\theta\theta'1} \geq p^*_{\theta\theta'2} \tag{65}$$

$$\mu_i \geq 0 \tag{66}$$

$$\mu_1(-x^*_{\theta\theta'}) + \mu_2(p^*_{\theta\theta'2} - p^*_{\theta\theta'1}) = 0 \tag{67}$$

5.7.2. Solution to PG&E Optimal Contract In this section, we discuss our empirical results, compare them against our theoretical results, and against the contracts which were offered by PG&E at the time of collecting the data.

Table 6 and Table 7 respectively present the optimal contract menu for first best, second best without battery and second best with battery offered to the two representative households we discussed in Section 5.5, which were $-\frac{\ln(A_\theta B_\theta)}{B_\theta - 1}$ equal to 1.86 and 2.1. As mentioned in Section 5.5, the difference in $-\frac{\ln(A_\theta B_\theta)}{B_\theta - 1}$ represents the difference in A_θ, which depends on household characteristics. The contracts are shown by total daily payment ($), demand in the off-peak time period (kWh), and demand in the peak time period (kWh). The peak time period is between 12pm and 6pm as per defined by PG&E for the time of data. Total agent surplus is calculated by the sum of agent surplus weighted by the percentage of users in each type.

We first discuss payments and electricity consumption allocations in tables 6 and 7. For second best without battery, price elastic Type 1 has binding IR constraint while all other types have one or more binding IC constraints. Also at least one of the types has a binding IC constraint against Type 1 contract. Consequently, price elastic household of Type 1 has similar payment to its first best while price inelastic household has reduced payment, for example Type 2 households payment reduces to almost 1/6th. Price elastic Type 1 however has more change in demand compared to the first best. These show that price inelastic households receive information rent, while price elastic households have more demand perturbation compared to the first best. Another observation for second best without battery is that all contracts have total daily payments close to the total daily payment of the most elastic type, Type 1, under the first best menu. This is due to binding IC constraints with Type 1 contract, and the fact that the proposed contract to each type provides close utility to the one the household would achieve if it had taken Type 1 contract. Next, comparing second best with battery to without battery, total payment, total demand, and allocation of demand in different periods change. For all types, demand decreases in peak while off-peak demand changes are in both directions. This indicates that although battery is not adopted, the option to purchase battery enforces the retailer to internalize some level of substitution in its contract offers. Again, each type has at least one binding IC constraint. This again results in close values of total payments.

Next we compare the welfare results of the three contract menus and validate our theoretical results in Section 4. In both Tables 6 and 7, the first best contract menu provides the highest social welfare and principle's surplus while giving the lowest agents surplus (equal to 0). Comparing second best without battery to the first best, the principle surplus decreases while the agents surplus increases. Moreover, all household types have increased surplus except type 1, the most elastic type, which has the binding IR constraint. Comparing second best with battery to without battery, the principle surplus decreases by 43% and 14% in the two cases of tables 6 and 7. But the agents' surplus goes up at heterogeneous ratios, from 2% to 400%. This validates our theoretical results. Finally, the social welfare also decreases changes at about 1-3% due to batteries.

Since there was no battery at the time of our data from PG&E, we can also compare the original retail tariff offered by PG&E with our second best contracts without battery. First, the daily average demand is higher under our contract. Second, households under our contract have average per kWh prices at around $1.5/kWh, while households under PG&E tariff pay between $0.1/kWh and $0.3/kWh, depending on the climate zone and tariff program they opt in. Higher demand and higher contract price in turn drives the retailer surplus in our contract. The fact that our contracts are not similar to PG&E tariffs can be because of the following reasons. PG&E was not a perfect rent-seeker at the time due to regulations on its profit margin, also PG&E has been limited in number and structure of tariffs it offers. However, in recent years, due to pressure from sources such as losses sustained in wildfires, PG&E has sought to increase its profit margin from current 10.25% to 16% [17]. Nowadays, PG&E is also able to offer a larger number of different tariffs including various types of ToU tariffs to its diverse population[18]. We expect as such changes take place, our contracts will become closer to practice. Nevertheless, studying the impact of such constraints on PG&E contracts is a future direction for this research.

Table 6 **Contracts for households with** $-\frac{\ln(A_\theta B_\theta)}{B_\theta - 1} = 1.86$ **for the nine types**

	First best	Second best w/o battery	Second best w/ battery
Contract 1	25.99 , 11.2 , 5.83	25.47 , 10.37 , 6.66	16.76 , 8.85 , 6.28
Contract 2	150.8 , 13.06 , 3.05	25.4 , 13.51 , 3.49	14.07 , 5.44 , 2.89
Contract 3	63.05 , 12.75 , 4.28	25.44 , 12.12 , 4.91	14.07 , 5.27 , 3.03
Contract 4	28.66 , 11.36 , 5.67	25.46 , 10.78 , 6.25	16.74 , 8.79 , 6.25
Contract 5	75.82 , 13.08 , 3.95	25.44 , 12.12 , 4.91	14.07 , 6.26 , 2.29
Contract 6	39.31 , 11.9 , 5.13	25.46 , 11.01 , 6.02	14.07 , 4.47 , 3.76
Contract 7	58.1 , 12.6 , 4.43	25.45 , 11.4 , 5.63	14.07 , 4.58 , 3.65
Contract 8	30.16 , 11.45 , 5.58	25.46 , 10.78 , 6.25	14.30 , 4.71 , 3.91
Contract 9	32.22 , 11.56 , 5.47	25.46 , 11.01 , 6.02	14.30 , 5.46 , 3.21
Agent surplus 1	0	0	8.25
Agent surplus 2	0	125.33	135.29
Agent surplus 3	0	37.41	46.77
Agent surplus 4	0	2.72	11.03
Agent surplus 5	0	50.22	59.74
Agent surplus 6	0	13.53	22.35
Agent surplus 7	0	32.44	41.72
Agent surplus 8	0	4.26	12.64
Agent surplus 9	0	6.36	14.86
Expected agent surplus	0	29.20	38.14
Principle surplus	53.96	24.43	13.91
Social welfare	53.96	53.63	52.05

[17] https://www.pge.com/pge_global/common/pdfs/your-account/your-bill/understand-your-bill/bill-inserts/2019/05-19-Cost-of-Capital.pdf

[18] https://www.pge.com/en_US/residential/rate-plans/how-rates-work/find-my-best-rate-plan.page?cid=ps_RPO_TOU2021_20210329_StartNow_Search__v1_TIME&gclid=Cj0KCQjw24qHBhCnARIsAPbdtlK9FSavrZ0EFYsuGjZAe0hGupS8iQMqtn9bEiyv7vXqJDYZWWXZCSMaAjnFEALw_wcB

Table 7 **Contracts for households with $-\frac{\ln(A_\theta B_\theta)}{B_\theta - 1} = 2.1$ for the nine types**

	First best	Second best w/o battery	Second best w/ battery
Contract 1	39.15 , 15.69 , 7.22	38.36 , 14.43 , 8.48	35.10 , 14.48 , 8.42
Contract 2	317.37 , 16.75 , 3.51	38.23 , 17.48 , 4.17	32.38 , 9.31 , 4.08
Contract 3	113.52 , 18.08 , 4.83	38.32 , 16.99 , 5.92	32.38 , 12.72 , 1.98
Contract 4	44.07 , 15.94 , 6.97	38.36 , 15.01 , 7.9	34.38 , 11.89 , 7.84
Contract 5	141.29 , 18.58 , 4.33	38.32 , 16.99 , 5.92	32.38 , 15.29 , 0.92
Contract 6	64.53 , 16.79 , 6.12	38.35 , 15.36 , 7.55	32.38 , 7.59 , 5.60
Contract 7	103 , 17.86 , 5.05	38.34 , 15.95 , 6.96	32.38 , 14.13 , 1.35
Contract 8	46.88 , 16.08 , 6.83	38.36 , 15.01 , 7.9	34.36 , 15.72 , 4.60
Contract 9	50.79 , 16.25 , 6.66	38.35 , 15.36 , 7.55	32.88 , 8.43 , 6.06
Agent surplus 1	0	0	3.27
Agent surplus 2	0	279.04	284.02
Agent surplus 3	0	74.91	79.32
Agent surplus 4	0	5.02	8.37
Agent surplus 5	0	102.74	107.30
Agent surplus 6	0	25.72	29.57
Agent surplus 7	0	64.37	68.70
Agent surplus 8	0	7.88	11.27
Agent surplus 9	0	11.84	15.35
Expected agent surplus	0	60.86	64.83
Principle surplus	98.33	36.99	31.99
Social welfare	98.33	97.85	96.82

6. Conclusion and Future Directions

We studied how the entrance of the electric battery providers for end-users will disturb the electricity retailers' business. We identified that when the retailer is social planner or has perfect information there is no change to the retailer pricing, and absent of arbitrage opportunity in wholesale market, batteries are not used by end-users. The retailer's welfare and social welfare will increase in those cases. For the case that the retailer is rent-seeker with imperfect information, we cast the problem into one of principal-agent with aftermarket for agents to substitute across products. We showed that there is no use of battery by end-users if there is no arbitrage opportunity, nor a dis-economy of scale in aftermarket, and the contract space is rich enough. This implies the aftermarket will not be realized, but the threat of its existence will force the retailer to change its offering contract. Furthermore, we showed behind-the-meter batteries will drive the retailer's welfare down, while the customers' and social welfare can be impacted in either directions.

We also empirically solved for optimal contracts for the case of PG&E in California using its hourly data from households consumption. Our structural model for households utility was based on exponential demand function mixed with AIDS model. We estimated the model in two stages, first using flat tiered tariff households with geographical price variation for estimating monthly elasticity, and second using time of use tiered tariff households with temporal price variation for estimating cross elasticities. We used unsupervised machine learning for estimation of the households' characteristics which impact their consumption but are not observed to retailer e.g. households lifestyles. The analysis of the optimal contracts validate our theoretical results. It showed behind-the-meter batteries result in up to 43% reduction in PG&E profit while increasing its customers' welfare heterogeneously 2% to 400%.

The future direction for this research includes identifying the retailer's optimal contract if one or some of the conditions (C1) to (C3) fail. Empirically, our techniques for the estimation of the household utility can be implemented for other electricity retailers. Using data from households electricity consumption with behind-the-meter battery is another complementary direction to this research.

Appendix A: Example

EXAMPLE 1. Consider two types $\theta \in \{\ell, h\}$ with demand of two time periods and additive utility functions of the following form.

$$U_\ell(d_1,d_2) = \kappa_{\ell 1} d_1^\alpha + \kappa_{\ell 2} d_2^\beta \tag{68}$$

$$U_h(d_1,d_2) = \kappa_{h1} d_1^\alpha + \kappa_{h2} d_2^\beta \tag{69}$$

Assume π_θ is the fraction of agents from type θ, and p_t is the price of principle's substitution at time t.

Case 1: Relaxing Condition (C2)- Set $\kappa_{\ell 1} = \kappa_{\ell 2} = \kappa_{h1} = 1$, $\kappa_{h2} > 1$, and $0 < \beta < 1$. Under such assumptions, type h always pretends to be ℓ. Thus, first best is never a feasible solution for the principle without agents' substitution. Adding agents' substitution, however, does not change this, because h has higher substitution and will have even higher utility given substitution compared to ℓ. To relax condition (C2), assume cost of substituting x from time period one to time period two is $\phi(x + x^2)$, which violates (C2).

We then solve the first best and second best problems under such settings. We provide numerical solutions for the following specifications of parameters. Note that these parameters satisfy (C1).

π_l	π_h	$\kappa_{\ell 1}$	$\kappa_{\ell 2}$	κ_{h1}	κ_{h2}	α	β	p_1	p_2	ϕ
0.4	0.6	1	1	1	1.65	0.4	0.3	0.2	0.3	0.2

Solving for the optimal contract with agent substitution numerically yields the following results where x_ℓ^* is positive.

$d_{\ell 1}$	$d_{\ell 2}$	d_{h1}	d_{h2}	t_ℓ	t_h	x_ℓ^*	x_h^*
2.8346	0.3838	3.1748	2.0450	2.2784	3.0615	0.1354	0

Case 2: Relaxing Condition (C3)- Consider the principle is limited to offering a single contract for these two types, so is forced to a pooling contract of $((d_1, d_2), t)$. The substitution cost for agents satisfies (C2) which is ϕx.

The principle provides supply from perfectly elastic wholesale market at prices p_1 and p_2 for the two time periods (so the substitution cost for the principle from time 1 to time 2 is $p_2 - p_1$). The cost of substitution for agents is $b^a(x) = \phi x > |p_2 - p_1|$. Consider $p := p_2 = p_1 >> \phi$ (this ensures the substitution is cheap enough for customers to fully substitute their demand from one period to the next). Also assume there are two types $\theta \in \{\theta_1, \theta_2\}$ with $\pi_{\theta_1} = 0.5$ (This ensures at the optimal contract both types opt in. If π_θ is small for one type, then it could be the case that the optimal contract violate IR for minority type). Assume the utility function of the two types are $U_{\theta_1}(d_1, d_2) = U_{\theta_2}(d_1, d_2) = (d_1 + d_2)^\beta$. We provide two cases: two-way substitution and one-way substitution. First consider a two-way substitution across two periods. For any offered demand profile (d_1, d_2), type θ_1 is going to substitute an amount determined by

$$x = \min\left\{d_2, (\frac{\beta}{\phi})^{\frac{1}{1-\beta}} - d_1, 0\right\} \tag{70}$$

similarly type θ_2 will substitute

$$x = \min\left\{d_1, (\frac{\beta}{\phi})^{\frac{1}{1-\beta}} - d_2, 0\right\} \tag{71}$$

For small ϕs the utility of agents will be

$$(d_1 + d_2)^\beta - \phi d_2 \tag{72}$$

$$(d_1 + d_2)^\beta - \phi d_1 \tag{73}$$

There are two cases, either both types opt in or IR for one type is violated. The best contract for which both types opt in is:

$$\max_{d_1, d_2, t} t - p(d_1 + d_2) \tag{74}$$

$$s.t. \qquad t \leq (d_1 + d_2)^\beta - \phi d_2 \tag{75}$$

$$t \leq (d_1 + d_2)^\beta - \phi d_1 \tag{76}$$

with solution being $d_1 = d_2 = \frac{1}{2}(\frac{\beta}{p})^{\frac{1}{1-\beta}}$, and $t = (\frac{\beta}{p})^{\frac{\beta}{1-\beta}} - \frac{\phi}{2}(\frac{\beta}{p})^{\frac{1}{1-\beta}}$. Both types then opt in, and get an amount of battery equal to $d_1 = d_2$. Alternatively, if one type opts out best contract is $d_1 = (\frac{\beta}{\phi})^{\frac{1}{1-\beta}}, d_2 = 0, t = d_1^\beta$ and profit is $0.5 \times (\frac{\beta}{\phi})^{\frac{\beta}{1-\beta}}$. No agent gets battery in this case. For small values of ϕ, the former profit dominates the latter and therefore, both firms opt in and get battery. The social welfare in this case is

$$(\frac{\beta}{p})^{\frac{\beta}{1-\beta}} - [c_1 + \frac{\phi}{2}](\frac{\beta}{p})^{\frac{1}{1-\beta}}. \tag{77}$$

Note that without contract limits, if the principle could offer two contracts, the optimal contract would be $d_1 = ((\frac{\beta}{p})^{\frac{1}{1-\beta}}, 0), d_2 = (0, (\frac{\beta}{p})^{\frac{1}{1-\beta}}), t_1 = t_2 = (\frac{\beta}{p})^{\frac{\beta}{1-\beta}}$. The social welfare would be

$$(\frac{\beta}{p})^{\frac{\beta}{1-\beta}} - c(\frac{\beta}{p})^{\frac{1}{1-\beta}}. \tag{78}$$

Therefore, the social welfare decreased with contract limit.

Next, if substitution is unidirectional from time 1 to time 2, for any offered demand profile (d_1, d_2), type θ_1 is not going to use substitution. But θ_2 is going to substitute an amount determined by

$$x = \min\left\{d_1, (\frac{\beta}{\phi})^{\frac{1}{1-\beta}} - d_2, 0\right\} \tag{79}$$

For small ϕs where $x = d_1$, his utility will be

$$(d_1 + d_2)^\beta - \phi d_1 \tag{80}$$

Therefore, the optimal contract will be

$$\max_{d_1,d_2,t} \pi_{\theta_1}(t - p(d_1 + d_2)) \times \mathbf{1}(t \le d_1^\beta) + (1 - \pi_{\theta_1})(t - p(d_1 + d_2)) \times \mathbf{1}(t \le (d_1 + d_2)^\beta - \phi d_1) \tag{81}$$

We consider two cases, both types are opting in or one IR being violated. With a violated IR, best contract is $d_1 = (\frac{\beta}{p})^{\frac{1}{1-\beta}}, d_2 = 0, t = d_1^\beta$ and profit is $0.5 \times [(\frac{\beta}{p})^{\frac{\beta}{1-\beta}} - p(\frac{\beta}{c})^{\frac{1}{1-\beta}}]$. No agent gets battery in this case. In the case both opt in, $d_1 = (\frac{\beta}{p})^{\frac{1}{1-\beta}}$, $d_2 = 0$, and $t = (d_1)^\beta - \phi d_1$. Profit is $t - pd_1 = (\frac{\beta}{p})^{\frac{\beta}{1-\beta}} - \phi(\frac{\beta}{c})^{\frac{1}{1-\beta}} - p(\frac{\beta}{p})^{\frac{1}{1-\beta}}$. Agent θ_2 substitutes $x = d_1$ with battery. For small values of ϕ, the second case is more profitable. The social welfare in this case would be

$$(\frac{\beta}{p})^{\frac{\beta}{1-\beta}} - [p + \frac{\phi}{2}](\frac{\beta}{p})^{\frac{1}{1-\beta}}. \tag{82}$$

which is less than the case with no contract limit.

Case 3: increase in agents' surplus and decrease in social welfare- Set

$$U_\ell(d_1, d_2) = d_1^\alpha \tag{83}$$

$$U_h(d_1, d_2) = \kappa_{h1} d_1^\alpha. \tag{84}$$

Let $\kappa_{h1} > 1$ so type h is the high type. If there is no agent substitution, the two type of users will never choose each other's contract. Therefore, second best without substitution is just the same as first best. If there exists agent substitution with substitution cost ϕx (ϕ small enough), the contract design problem of second best without substitution is

$$\max_{d_{\ell 1},d_{h1},t_\ell,t_h} t_\ell + t_h - p_1 d_{\ell 1} - p_1 d_{h1} \tag{85}$$

$$s.t. \qquad t_\ell \le d_{\ell 1}^\alpha, IR_\ell \tag{86}$$

$$t_h \le \kappa_{h1} d_{h1}^\alpha, IR_h \tag{87}$$

$$\kappa_{h1} d_{\ell 1}^\alpha - t_\ell \le \kappa_{h1} d_{h1}^\alpha - t_h, IC_h \tag{88}$$

Substituting the binding constraints into the objective function

$$\max_{d_{\ell 1},d_{h1}} (2 - \kappa_{h1}) d_{\ell 1}^\alpha - p_1 d_{\ell 1} + \kappa_{h1} d_{h1}^\alpha - p_1 d_{h1} \tag{89}$$

It is obvious that with substitution agent surplus increases according to the IC constraint in (88) and agent surplus being 0 without substitution. Social welfare decreases with substitution because $2 - \kappa_{h1} \neq 1$ and thus deviates from first best in (89).

Case 4: decrease in agents' surplus and social welfare-

$$U_\ell(d_1, d_2) = d_1^\alpha \tag{90}$$

$$U_h(d_1, d_2) = \kappa_{h1} d_1^\alpha + \kappa_{h2} d_2^\beta \tag{91}$$

Suppose $\kappa_{h1} > 1$, then the binding IC is always IC_h here. Assume $p_1 = p_2 = p$ and agent substitution cost ϕx. The contract design problem of second best without substitution

$$\max_{d_\ell, d_h, t_\ell, t_h} t_\ell + t_h - pd_{\ell 1} - p(d_{h1} + d_{h2}) \tag{92}$$

$$s.t. \quad t_\ell \le d_{\ell 1}^\alpha, IR_\ell \tag{93}$$

$$t_h \le \kappa_{h1} d_{h1}^\alpha + \kappa_{h2} d_{h2}^\beta, IR_h \tag{94}$$

$$\kappa_{h1} d_{\ell 1}^\alpha - t_\ell \le \kappa_{h1} d_{h1}^\alpha + \kappa_{h2} d_{h2}^\beta - t_h, IC_h \tag{95}$$

The binding constraints are IR_ℓ and IC_h. Substituting the binding constraints into the objective function

$$\max_{d_\ell, d_h} (2 - \kappa_{h1}) d_{\ell 1}^\alpha - pd_{\ell 1} + \kappa_{h1} d_{h1}^\alpha + \kappa_{h2} d_{h2}^\beta - p(d_{h1} + d_{h2}) \tag{96}$$

The solution for d_h is the same as under first best, and the solution for d_ℓ is

$$d_{\ell 1}^{SB/wo} = [\frac{p}{\alpha(2 - \kappa_{h1})}]^{1/(\alpha-1)} \tag{97}$$

Now we consider second best with substitution. We assume $\kappa_{h2} = \kappa_{h1}$ and $\beta = \alpha$. If ϕ is small enough, we will have

$$V_h(d_\ell) = \kappa_{h1}(d_{\ell 1}/2)^\alpha + \kappa_{h1}(d_{\ell 1}/2)^\alpha \tag{98}$$

$$= 2\kappa_{h1}(1/2^\alpha)(d_{\ell 1})^\alpha \tag{99}$$

The IC_h constraint defined in (92) now becomes

$$2\kappa_{h1}(1/2^\alpha) d_{\ell 1}^\alpha - t_\ell \le \kappa_{h1} d_{h1}^\alpha + \kappa_2 d_{h2}^\beta - t_h, IC_h \tag{100}$$

The contract design problem now becomes

$$\max_{d_\ell, d_h} (2 - 2\kappa_{h1}(1/2^\alpha)) d_{\ell 1}^\alpha - pd_{\ell 1} + \kappa_{h1} d_{h1}^\alpha + \kappa_{h2} d_{h2}^\beta - p(d_{h1} + d_{h2}) \tag{101}$$

The solution for d_ℓ is

$$d_{\ell 1}^{SB/w} = [\frac{p}{\alpha(2 - 2\kappa_{h1}(1/2^\alpha))}]^{1/(\alpha-1)} \tag{102}$$

Comparing (101) and (96), **it is clear social welfare decreases with substitution** given $2\kappa_{h1}(1/2^\alpha) > \kappa_{h1}$ and $\kappa_{h1} > 1$. Next, we check agent surplus. For both with and without substitution, agent surplus of type ℓ $as_l = 0$. Since IC_h binds in both conditions, the agent surplus can be given by

$$as^{SB/wo} = (\kappa_{h1} - 1)[\frac{p}{\alpha(2 - \kappa_{h1})}]^{\alpha/(\alpha-1)} \tag{103}$$

$$as^{SB/w} = (2\kappa_{h1}(1/2^\alpha) - 1)[\frac{p}{\alpha(2 - 2\kappa_{h1}(1/2^\alpha))}]^{\alpha/(\alpha-1)} \tag{104}$$

Comparing these two values is equivalent to comparing

$$t(1 - t)^{\alpha/(1-\alpha)} \tag{105}$$

when t is equal to $\kappa_{h1} - 1$ and $2\kappa_{h1}(1/2^\alpha) - 1$, respectively. As $2\kappa_{h1}(1/2^\alpha) - 1 > \kappa_{h1} - 1$, we only need to check the monotonicity of (105). Let $\alpha = 0.5$, (105) becomes $t(1-t)$, which is monotonically decreasing when $t \ge 0.5$. Therefore, if $\kappa_{h1} \ge 1.5$, the agent surplus decreases with substitution.

Case 5: increase agents surplus and social welfare- Set

$$U_\ell(d_1, d_2) = \kappa_{\ell 1} d_1^\alpha + \kappa_{\ell 2} d_2^\beta, \tag{106}$$

$$U_h(d_1, d_2) = \kappa_{h2} d_2^\beta. \tag{107}$$

We want to make a case where type h is always the high type here no matter with or without substitution (this can be satisfied by making κ_{h2} large). Again assume $p_1 = p_2 = p$ and agent substitution cost ϕx. If this is true, then the binding IC is always IC_h. The contract design problem of second best without substitution

$$\max_{d_\ell, d_h, t_\ell, t_h} t_\ell + t_h - p(d_{\ell 1} + d_{\ell 2}) - p d_{h2} \tag{108}$$

$$s.t. \qquad t_\ell \le \kappa_{\ell 1} d_{\ell 1}^\alpha + \kappa_{\ell 2} d_{\ell 2}^\beta, IR_\ell \tag{109}$$

$$t_h \le \kappa_{h2} d_{h2}^\beta, IR_h \tag{110}$$

$$\kappa_{h2} d_{\ell 2}^\beta - t_\ell \le \kappa_{h2} d_{h2}^\beta - t_h, IC_h \tag{111}$$

The binding constraints are IR_ℓ and IC_h. Substituting the binding constraints into the objective function

$$\max_{d_\ell, d_h} 2\kappa_{\ell 1} d_{\ell 1}^\alpha - p d_{\ell 1} + (2\kappa_{\ell 2} - \kappa_2) d_\ell^{2\beta} - p d_\ell^2 + \kappa_{h2} d_{h2}^\beta - p d_{h2} \tag{112}$$

The solution for d_h is the same as under first best, and the solution for d_ℓ, given κ_{h2} being large, is

$$d_{\ell 1}^{SB/wo} = [\frac{p}{2\alpha\kappa_{\ell 1}}]^{1/(\alpha-1)} \tag{113}$$

$$d_{\ell 2}^{SB/wo} = 0 \tag{114}$$

Now we consider second best with substitution. We assume $\beta = \alpha$. If ϕ is small enough, we will have

$$V_h(d_\ell) = \kappa_{h2}(d_{\ell 1} + d_{\ell 2})^\alpha \tag{115}$$

The IC_h constraint defined in (108) now becomes

$$\kappa_{h2}(d_{\ell 1} + d_{\ell 2})^\alpha - t_\ell \le \kappa_{h2} d_{h2}^\alpha - t_h, IC_h \tag{116}$$

The contract design problem now becomes

$$\max_{d_\ell, d_h} 2\kappa_{\ell 1} d_{\ell 1}^\alpha - p d_{\ell 1} + 2\kappa_{\ell 2} d_{\ell 2}^\alpha - p d_{\ell 2} - \kappa_{h2}(d_{\ell 1} + d_{\ell 2})^\alpha + \kappa_{h2} d_{h2}^\alpha - p d_{h2} \tag{117}$$

It is not hard to observe that the solution for $d_{\ell 2}$ is still

$$d_{\ell 2}^{SB/w} = 0 \tag{118}$$

Given this observation, (117) can be rewritten as

$$\max_{d_\ell, d_h} (2\kappa_{\ell 1} - \kappa_{h2}) d_{\ell 1}^\alpha - p d_{\ell 1} + \kappa_{h2} d_{h2}^\alpha - p d_{h2} \tag{119}$$

Now the solution for $d_{\ell 1}$ is

$$d_{\ell 1}^{SB/w} = [\frac{p}{\alpha(2\kappa_{\ell 1} - \kappa_{h2})}]^{1/(\alpha-1)} \tag{120}$$

Now the problem becomes which $d_{\ell 1}$ in (120) or (113) gives higher social welfare. Note that both of them deviate from the first best case where

$$d_{\ell 1}^{FB} = [\frac{p}{\alpha\kappa_{\ell 1}}]^{1/(\alpha-1)} \tag{121}$$

Therefore, if we let $\kappa_{\ell 2}$ to be small enough, and let κ_{h2} to be greater than but close to $\kappa_{\ell 1}$ (these are to ensure h being the high type with and without battery), the solution of second best with battery will be close to first best, and thus has higher social welfare than second best without battery. Note that once having higher social welfare, it also means having higher agent surplus because principal surplus decreases with battery.

Appendix B: PG&E Tariff Programs

Nonlinear Electricity Pricing

Both E1 and E7 users are under non-linear pricing structure. The marginal electricity price is a step function of monthly consumption relative to a "baseline" consumption level. The marginal price equals the first tier rate up to 100% of the baseline, the second tier rate up to 130%, the third tier rate up to 200%, the fourth tier rate up to 300%, and the fifth tier rate over 300% of the baseline. The electricity consumed above the baseline allowance is billed at higher price depending on the related tier. The baseline differs by climate zones.

Time-of-Use (ToU) Electricity Pricing

The ToU users faced a combination of tiered and time-varying price. Specifically, on-peak and off-peak usage is assigned to tiers on a pro-rated basis. For example, if twenty percent of a household's usage is in the on-peak period, then twenty percent of the total usage in each tier will be treated as on-peak usage and eighty percent of the total usage in each tier will be treated as off-peak usage. The peak time periods are 12:00 noon to 6:00 p.m. Monday through Friday with all the other hours being off-peak periods.

Baseline Allowance The baseline allowance set by PG&E varies per climate zone, end use type, and season. Households who are eligible for the end use type of Medical Baseline receive an additional allotment of electricity at approximately 500 kWh per month added to its baseline allowance; this will shift the baseline in the first tier but also all other baselines which are determined as a ration of the first. PG&E changed the baseline allowance once during the consumption time of our observed users. We listed the baseline allowance before and after the change in Table 8 and Table 9, respectively. The baseline allowance in all climate zones decreases except for Zone V. Tiered rates of CARE for both E1 and E7 are provided in Table 6.

Table 8 Electric baseline (kWh/day) from May 1, 2008 to June 19, 2011 based on climate zones, end use type (H: all-electric, B: basic), and season.

Zone	Winter (H)	Winter (B)	Summer (H)	Summer (B)
P	35.5	12.9	20.1	16.5
Q	22.9	12.6	11.1	8.3
R	32.6	12.3	23.2	18.1
S	32.0	12.7	20.1	16.5
T	20.2	9.8	11.1	8.3
V	27.5	11.1	16.5	9.6
W	29.2	11.4	27.3	19.4
X	22.9	12.6	12.2	12.1
Y	30.9	13.3	15.0	12.1
Z	31.5	11.6	12.8	12.2

Table 9 Electric baseline (kWh/day) from June 20, 2011 to July 31, 2014 based on climate zones, end use type (H: all-electric, B: basic), and season.

Zone	Winter (H)	Winter (B)	Summer (H)	Summer (B)
P	33.9	12.7	18	15.3
Q	19.3	11.7	9.1	7.5
R	30.2	11.7	20.9	17.1
S	28.6	12	18	15.3
T	16.8	9.1	9.1	7.5
V	33.4	13.6	19.4	12
W	22.8	10.9	23.5	18.5
X	19.3	11.7	10.3	11
Y	30.7	13.2	14.1	11.7
Z	28.5	12	16.9	13

Table 10 shows the residential electric rates of tiered users and ToU users[19]. In addition to the difference caused by tiers, ToU prices also vary by season and by peak/off-peak periods. Households satisfying certain income requirements can apply to be enrolled in the CARE program which provides lower prices for households.

Table 10 Tariff rates in $/kWh from June 1, 2010 to December 31, 2010. E1: residential tiered rate, EL1: low income residential CARE E1 rate, E7: time-of-use rate (peak: 12pm to 6pm weekdays; off-peak: all the other hours), EL7: CARE E7 time-of-use rate

Tariff	Season	Period	Tier 1	Tier 2	Tier 3	Tier 4	Tier 5
E1	-	-	0.11877	0.13502	0.29062	0.40029	0.40029
EL1	-	-	0.08316	0.09563	0.09563	0.09563	0.0956
E7[a, b]	Winter	Peak	0.11936	0.11936	0.27523	0.38463	0.38463
E7	Winter	Off-Peak	0.09318	0.09318	0.24905	0.35845	0.35845
E7	Summer	Peak	0.30631	0.30631	0.46218	0.57158	0.57158
E7	Summer	Off-Peak	0.09003	0.09003	0.2459	0.3553	0.3553
EL7[c]	Winter	Peak	0.10472	0.10472	0.10472	0.10472	0.10472
EL7	Winter	Off-Peak	0.07966	0.07966	0.07966	0.07966	0.07966
EL7	Summer	Peak	0.28372	0.28372	0.28372	0.28372	0.28372
EL7	Summer	Off-Peak	0.07664	0.07664	0.07664	0.07664	0.07664

[a] An additional meter charge is applied to E7 users at $0.11532/meter·day

[b] A baseline credit is applied to E7 users at $0.01679/kWh

[c] A baseline credit is applied to EL7 users at $0.01559/kWh

Households Demographics Table 11 shows the demographics and average consumption of households in different climate zones.

[19] We only show the rates from June 1, 2010 to December 31, 2010. Rates in the rest periods can be found at https://www.pge.com/tariffs/electric.shtml

Table 11 Average household demographics and electricity consumption across climate zones.

Zone	P	R	S	T	W	X	Y	Z
Income	43622	41533	55366	55227	39311	74253	44523	65970
Household size	2.35	3.21	2.97	2.61	3.32	2.8	2.27	1.97
Average daily electricity use (kWh)								
Jan	19.9	17.3	18.76	14.31	15.7	17.58	13.6	10.91
Feb	19.84	16.38	17.90	13.85	14.84	16.79	14.19	12.10
Mar	18.81	14.93	16.81	13.22	13.53	15.83	13.31	10.86
Apr	17.44	16.03	17.22	12.37	15.46	15.34	12.54	8.95
May	14.40	16.35	16.38	10.23	17.39	13.01	10.32	6.34
Jun	19.07	24.78	23.83	11.55	26.94	16.03	12.15	7.15
Jul	21.73	29.42	25.86	10.83	31.36	15.41	13.04	8.63
Aug	25.34	33.71	29.09	11.28	36.60	16.52	14.60	8.14
Sep	19.18	25.88	21.70	11.41	28.07	14.88	11.87	7.38
Oct	16.70	17.16	17.86	11.73	17.41	15.12	11.60	7.49
Nov	19.71	16.90	18.59	14.04	15.40	17.30	14.24	9.68
Dec	21.10	18.73	20.01	14.72	17.39	18.40	14.67	12.51

Appendix C: Second stage Regression Results for ToU Users

Table 12 shows the second stage results.

Table 12 Second stage regression coefficients

Hour	γ_{hP}	β_h^{AIDS}
1	-0.063 (0.016)	-0.006 (0.002)
2	-0.053 (0.016)	-0.006 (0.002)
3	-0.054 (0.016)	-0.006 (0.002)
4	-0.066 (0.017)	-0.008 (0.002)
5	-0.063 (0.017)	-0.013 (0.002)
6	-0.055 (0.017)	-0.012 (0.002)
7	-0.033 (0.016)	-0.007 (0.002)
8	-0.007 (0.017)	-0.004 (0.003)
9	-0.02 (0.017)	-0.001 (0.003)
10	-0.029 (0.017)	0.005 (0.003)
11	-0.055 (0.013)	0.016 (0.002)
12	-0.051 (0.01)	0.02 (0.002)
13	0.031 (0.024)	0.038 (0.004)
14	-0.086 (0.025)	0.026 (0.003)
15	-0.219 (0.033)	0.011 (0.003)
16	-0.294 (0.041)	-0.004 (0.004)
17	-0.265 (0.042)	-0.007 (0.005)
18	-0.17 (0.039)	-0.022 (0.006)
19	0.053 (0.013)	-0.009 (0.002)
20	-0.049 (0.014)	-0.006 (0.002)
21	-0.137 (0.02)	-0.001 (0.002)
22	-0.147 (0.021)	0.001 (0.002)
23	-0.1 (0.018)	0 (0.002)
24	-0.075 (0.016)	-0.004 (0.002)

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